S1 June 2018 (MA)

Q1a) $F(3) = P(X=2) = a = \boxed{0.2}$

$F(6) = P(X=2) + P(X=4) = a + b = 0.8$

$\therefore b = \boxed{0.6}$

$a + b + 0.1 + c = 1$

$c = 1 - 0.1 - 0.6 - 0.2 = \boxed{0.1} = c$

b) $F(7) = P(X \leq 7) = 0.1 + b + a = \boxed{0.9}$

Q2a) $(3-6) \rightarrow \text{width} = 2 = (4 \times \frac{1}{2})$

$(11-15) \rightarrow \text{width} = \boxed{2.5}\text{ cm} = (5 \times \frac{1}{2})$

Area $= k \times f$.

$(2 \times 9.5) = k(38) \quad \therefore k = \frac{1}{2}$ } for (3-6)

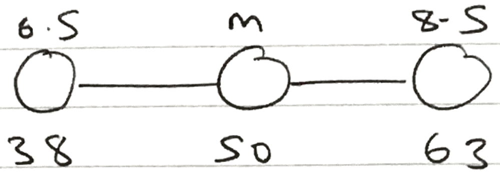
so Area for $(11-15) = \frac{1}{2} \times 12 = 6$

$\rightarrow 2.5 \times h = 6$

$\therefore h = \frac{6}{2.5} = \boxed{2.4}$ cm

b) $\frac{n}{2} = \frac{100}{2} = 50$

so median lies in 2^{nd} interval.



$$\frac{m-6.5}{8.5-6.5} = \frac{50-38}{63-38}$$

$$m - 6.5 = 2\left(\frac{12}{25}\right)$$

$$m = 6.5 + \frac{24}{25} = \boxed{7.46}$$

c) $$\text{mean} = \frac{\sum fx}{\sum f} = \frac{38(4.5) + 25(7.5) + 18(9.5) + 12(13) + 7(18)}{100}$$

$$= \frac{811.5}{100} = \boxed{8.12} \ (3sf)$$

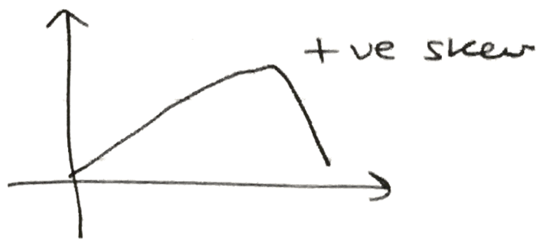
d) $$s.d = \sqrt{\frac{\sum fx^2}{\sum f} - (8.12)^2} = \sqrt{\frac{8096.25}{100} - (8.12..)^2}$$

$$= \boxed{3.89}$$

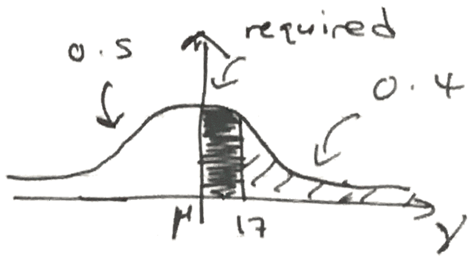
e) $\dfrac{3(8.12-7.46)}{3.89} = 0.5055.. = \boxed{0.51}$

f) The skewness are different for Mondays and Fridays.. $\left(\begin{matrix} -0.22 < 0 \\ 0.51 > 0 \end{matrix}\right)$

So friday will have longer delays since - ve skew



Q3a) $P(\mu < Y < 17) = 0.6 - 0.5$
$= \boxed{0.1}$



b) $P(\mu - \sigma < Y < 17) = P(Y < 17) - P(Y < \mu - \sigma)$

$= 0.6 - P\left(Z < \frac{-\sigma}{\sigma}\right)$

$= 0.6 - P(Z < -1)$

$= 0.6 - [1 - P(Z < 1)]$

$= \boxed{0.4413}$

$r + y = 63$

Q4a) $P(\text{green}) = P(G) + P(RG) + P(YG)$

$$= \frac{1}{64} + \left(\frac{63-y}{64}\right)\left(\frac{1}{63}\right) + \left(\frac{63-r}{64}\right)\left(\frac{1}{63}\right)$$

$$= \frac{63+63+63-r-y}{64 \times 63}$$

$$= \frac{189-(r+y)}{64 \times 63} = \frac{189-(63)}{64 \times 63}$$

$$= \boxed{\frac{1}{32}}$$

b) $P(RR) = \left(\frac{63-y}{64}\right)\left(\frac{62-y}{63}\right) = \frac{5}{84}$

$\underline{y = 63 - r}$: $\left(\frac{63-63+r}{64}\right)\left(\frac{62-63+r}{63}\right) = \frac{5}{84}$

$$r(r-1) = \frac{5}{84} \times 64 \times 63$$

$$r^2 - r = 240$$

hence $r^2 - r - 240 = 0$

c) $a = 1$, $b = -1$, $c = -240$ $\Bigg\}$ $r = \dfrac{1 \pm \sqrt{1 - 4(-240)}}{2}$

$$r = \frac{1 \pm \sqrt{961}}{2}$$

$$r = \frac{1 \pm 31}{2}$$

now $r > 0$ so $r = \dfrac{1+31}{2}$

$$= \boxed{16}$$

d) $P(\text{both red} \mid \text{at least one red}) = \dfrac{P(\text{both red} \cap \text{one is red})}{P(\text{at least one red})}$

$$= \frac{P(\text{both red})}{P(\text{at least one red})}$$

$$= \frac{P(RR)}{1 - P(\text{no red})}$$

$$= \frac{\frac{5}{84}}{\frac{37}{84}}$$

← we are given this.

$$P(\geq 1\text{ red}) = P(R) + P(R'R)$$

$$= \frac{16}{64} + \frac{48}{64} \times \frac{16}{63} = \frac{37}{84}$$

//

$$= \boxed{\frac{5}{37}}$$

Q5 a) $E(X) \left(= -1b + 0a + 2a + 4a + 5b\right)$

$= 2$ because the distribution is symmetric about $x = 2$

b) $-b + 2a + 4a + 5b = 2$

$6a + 4b = 2$

$$\boxed{3a + 2b = 1}$$

c) $E(X^2) = b + 4a + 16a + 25b$

$= 26b + 20a$

$Var(X) = E(X^2) - [E(X)]^2$

$\Rightarrow Var(X) + [E(X)]^2 = E(X^2)$

//

d) from (b), $b = \frac{1}{2} - \frac{3}{2}a$

$$20a + 26\left(\frac{1}{2} - \frac{3}{2}a\right) = 11.1$$

$$20a + 13 - 39a = 11.1$$

$$19a = 13 - 11.1 = 1.9$$

$$a = \frac{1.9}{19} = \boxed{0.1}$$

$$\therefore b = \frac{1}{2} - \frac{3}{2}(0.1) = \boxed{0.35}$$

ei) $E(Y) = E(10-3X) = -3E(X) + 10$
$= -3(2) + 10 = \boxed{4}$

ii) $Var(Y) = Var(10-3X) = 3^2 Var(X) = 9(7.1) = \boxed{63.9}$

f) $P(Y > X) = P(10-3X > X) = P(10 > 4X)$

$$= P\left(\frac{5}{2} > X\right)$$

$$= P(X < 2.5)$$

$$= P(X=2) + P(X=0) + P(X=-1)$$

$$= 2a + b$$
$$= 2(0.1) + 0.35 = \boxed{0.55}$$

Q6a) $S_{th} = 31070 - \frac{(61)(6370)}{8} = \boxed{-17501.25}$

$S_{tt} = 693 - \frac{(61)^2}{8} = \boxed{227.875}$

b) $r \simeq -1$ $\therefore$ strong -ve correlation suggested. so data supports the linear model.

c) $b = \frac{S_{th}}{S_{hh}}$.

$r = \frac{S_{th}}{\sqrt{S_{hh} \times S_{tt}}} = -0.985$

$\therefore \sqrt{S_{hh} \times 227.875} = \frac{-17501.25}{-0.985}$

$S_{hh} \times 227.875 = \left(\frac{17501.25}{0.985}\right)^2$

$S_{hh} = \frac{\left(\frac{17501.25}{0.985}\right)^2}{227.875} = \underline{\underline{1385380.26}}$

$\therefore b = \frac{-17501.25}{1385380.26} = \underline{\underline{-0.013}}$

$a = \bar{t} - b\bar{h} = \frac{61}{8} - (-0.013)\left(\frac{6370}{8}\right)$

$= 17.68\ldots = 17.7$

so $t = 17.7 - 0.013h$

$\hookrightarrow \boxed{t = 17.7 - 0.0126h}$

3sf.

d) temperature at sea level = 17.7°C

e) $150 \times b = \boxed{1.89°}$

Q7a) $W \sim N(140, 40^2)$

$P(W < 92) = P\left(Z < \frac{92-140}{40}\right) = P(Z < -1.2)$

$= 1 - P(Z < 1.20) = 1 - 0.8849$

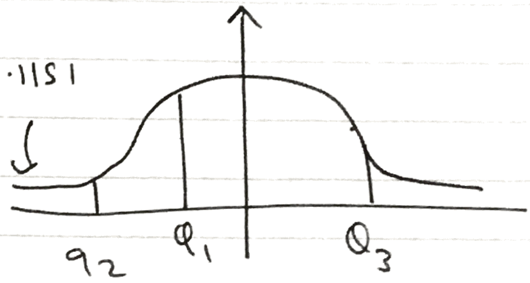
$= 0.1151$

so % $\rightarrow \boxed{11.5\%}$

b) $P(W>92) = 1 - 0.1151 = 0.8849$

$0.25 \times 0.8849 = \boxed{0.2212}$

c) $P(W<Q_1) = 0.1151 + 0.2212$



$P(W<Q_1) = 0.3363$

$P\left(Z < \frac{Q_1 - 140}{40}\right) = 0.3363$

$P\left(Z < \frac{140 - Q_1}{40}\right) = 0.6637$

$\therefore \frac{140 - Q_1}{40} \simeq 0.42$

$Q_1 \simeq 140 - 40(0.42) \simeq 123.2$

so $\boxed{Q_1 = 123g}$

d) $P(W<Q_1) = 0.2212$

$P(W>Q_3) = 0.2212$

$P(Q_1<W<Q_3) = 1 - 2(0.2212) = 0.5576$

$\therefore P(\text{required}) = 6\times(0.2212)(0.2212)(0.5576)$

$=$

d) Betty only picks sold potatoes.

$\therefore P(w < Q_1) = P(w > Q_3) = \frac{1}{4}$

$P(Q_1 < w < Q_3) = \frac{1}{2}$

$P(\text{required}) = 6 \times \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \boxed{\frac{3}{16}}$

There are 6 different ways these potatoes could be arranged.

(ie) the first could be $< Q_1$, and the second between Q_1 and Q_3 or maybe the first is $> Q_3$ and the second is $< Q_1$. etc