S1 S14 (R)

**1.** The discrete random variable $X$ has probability distribution

| $x$ | $-4$ | $-2$ | 1 | 3 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.4 | $p$ | 0.05 | 0.15 | $p$ |

(a) Show that $p = 0.2$

**(2)**

Find

(b) $E(X)$

**(2)**

(c) $F(0)$

**(1)**

(d) $P(3X + 2 > 5)$

**(2)**

Given that $\text{Var}(X) = 13.35$

(e) find the possible values of $a$ such that $\text{Var}(aX + 3) = 53.4$

**(2)**

a) $\Sigma P = 1 \quad 0.4 + p + 0.05 + 0.15 + p = 1 \Rightarrow 2p = 0.4 \quad p = \underline{0.2}$ 2

b) $E(X) = -1.6 - 0.4 + 0.05 + 0.45 + 1 = \underline{-0.5}$ 2

c) $F(0) = P(x \leq 0) = 0.4 + 0.2 = \underline{0.6}$ 2

d)

| $3x+2$ | $-10$ | $-4$ | 5 | 11 | 17 |
|---|---|---|---|---|---|
| $x$ | $-4$ | $-2$ | 1 | 3 | 5 |
| $p$ | 0.4 | 0.2 | 0.05 | 0.15 | 0.2 |
| | | | | $>5$ ✓ | $>5$ ✓ |

$P(3X+2>5) = \underline{0.35}$ 2

e) $\text{Var}(aX+3) = a^2 V(X) = 53.4$

$\therefore a^2 = \dfrac{53.4}{13.35} \quad \therefore a^2 = 4$

$\therefore a = \pm 2$

2

**2.** The discrete random variable $X$ has probability distribution

$$P(X = x) = \frac{1}{10} \qquad x = 1, 2, 3, \ldots 10$$

(a) Write down the name given to this distribution.

**(1)**

(b) Write down the value of

(i) $P(X = 10)$

(ii) $P(X < 10)$

**(2)**

The continuous random variable $Y$ has the normal distribution $N(10, 2^2)$

(c) Write down the value of

(i) $P(Y = 10)$

(ii) $P(Y < 10)$

**(2)**

a) Discrete Uniform distribution

b) i) $P(X=10) = \frac{1}{10}$ ii) $P(X<10) = \frac{9}{10}$

c) i) $P(Y=10)=0$

ii) $P(Y<10) \approx P(Z < \frac{10-10}{2}) = P(Z<0) = \underline{\underline{0.5}}$

3. A large company is analysing how much money it spends on paper in its offices every year. The number of employees, $x$, and the amount of money spent on paper, $p$ (£ hundreds), in 8 randomly selected offices are given in the table below.

| $x$ | 8 | 9 | 12 | 14 | 7 | 3 | 16 | 19 |
|---|---|---|---|---|---|---|---|---|
| $p$ (£ hundreds) | 40.5 | 36.1 | 30.4 | 39.4 | 32.6 | 31.1 | 43.4 | 45.7 |

(You may use $\sum x^2 = 1160$ $\quad \sum p = 299.2$ $\quad \sum p^2 = 11\,422$ $\quad \sum xp = 3449.5$)

(a) Show that $S_{pp} = 231.92$ and find the value of $S_{xx}$ and the value of $S_{xp}$

**(5)**

(b) Calculate the product moment correlation coefficient between $x$ and $p$.

**(2)**

The equation of the regression line of $p$ on $x$ is given in the form $p = a + bx$.

(c) Show that, to 3 significant figures, $b = 0.824$ and find the value of $a$.

**(4)**

(d) Estimate the amount of money spent on paper in an office with 10 employees.

**(2)**

(e) Explain the effect each additional employee has on the amount of money spent on paper.

**(1)**

Later the company realised it had made a mistake in adding up its costs, $p$. The true costs were actually half of the values recorded. The product moment correlation coefficient and the equation of the linear regression line are recalculated using this information.

(f) Write down the new value of

(i) the product moment correlation coefficient,

(ii) the gradient of the regression line.

**(2)**

a) $S_{pp} = \Sigma p^2 - (\Sigma p)^2 \div n = 11422 - 299.2^2 \div 8 = 231.92$

$S_{xx} = 1160 - 88^2 \div 8 = 192$ $\quad \Sigma x = 88$

$S_{xp} = 3449.5 - (88)(299.2) \div 8 = 158.3$

b) PMCC, $r = \dfrac{158.3}{\sqrt{192 \times 231.92}} = 0.7502 \quad (0.75)$

3c) $y = a + bx$ $\quad b = \frac{Sxy}{Sxx} \rightarrow \frac{Sxp}{Sxx} = \frac{158.3}{192} = 0.824 \ (3sf)$

$p = a + bx$

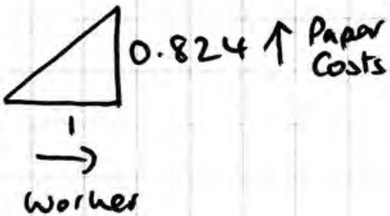
$$a = \bar{y} - b\bar{x} \Rightarrow a = \bar{p} - b\bar{x} = \left(\frac{299.2}{8}\right) - 0.824\ldots \times \left(\frac{192}{8}\right)$$

$$a = 17.6 \ (3sf)$$

$$\therefore \quad p = 17.6 + 0.824x$$

d) $x = 10 \qquad p = 17.6 + 0.824(10) = 25.84 \quad \therefore £2584$

e)




each additional worker results in an extra £82.40 spent on paper

f) i) PMCC will be unchanged.

ii) the gradient will be halved.

**4.** $A$ and $B$ are two events such that

$$P(B) = \frac{1}{2} \qquad P(A \mid B) = \frac{2}{5} \qquad P(A \cup B) = \frac{13}{20}$$

(a) Find $P(A \cap B)$.

**(2)**

(b) Draw a Venn diagram to show the events $A$, $B$ and all the associated probabilities.

**(3)**

Find

(c) $P(A)$

**(1)**

(d) $P(B \mid A)$

**(2)**

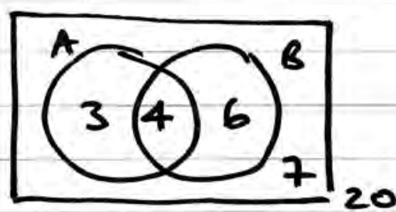
(e) $P(A' \cap B)$

**(1)**

a) $P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$

2

b) $P(B) = \frac{10}{20}$ $P(A \cap B) = \frac{4}{20}$




d) $P(B|A) = \frac{4}{7}$ c) $P(A) = \frac{7}{20}$

e) $P(A' \cap B) = \frac{6}{20}$

**5.** The table shows the time, to the nearest minute, spent waiting for a taxi by each of 80 people one Sunday afternoon.

| Waiting time (in minutes) | Frequency | | cw | fd |
|---|---|---|---|---|
| | | 0 | | |
| 2–4 | 15 | 15 | 3 | 5 |
| 5–6 | 9 | 24 | 2 | 4.5 |
| 7 | 6 | 30 | 1 | 6 $\rightarrow \div 4$ |
| 8 | 24 | 54 | 1 | 24 $= 6\text{cm}$ |
| 9–10 | 14 | 68 | 2 | 7 |
| 11–15 | 12 | 80 | 5 | 2.4 |

(a) Write down the upper class boundary for the 2–4 minute interval.
**(1)**

A histogram is drawn to represent these data. The height of the tallest bar is 6 cm.

(b) Calculate the height of the second tallest bar.
**(3)**

(c) Estimate the number of people with a waiting time between 3.5 minutes and 7 minutes.
**(2)**

(d) Use linear interpolation to estimate the median, the lower quartile and the upper quartile of the waiting times.
**(4)**

(e) Describe the skewness of these data, giving a reason for your answer.
**(2)**

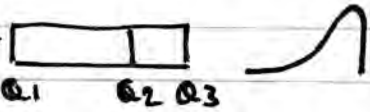
a) $\underline{4.5}$   b) $\frac{7}{4} = \underline{1.75}$   c) $\boxed{1}\,5 = (5) + (9) + \boxed{\tfrac{1}{2}}\,6 + (3)$
3.5  4.5   6.5  7
$= \underline{17}$

d) $\frac{1}{4}n = 20$  4.5 $Q_1$ 6.5 / 15 20 24  $\frac{Q_1 - 4.5}{2} = \frac{5}{9}$ $\therefore Q_1 = \underline{5.61}$

$\frac{1}{2}n = 40$  7.5 $Q_2$ 8.5 / 30 40 54  $\frac{Q_2 - 7.5}{1} = \frac{10}{24} \Rightarrow Q_2 = \underline{7.92}$

$\frac{3}{4}n = 60$  8.5 $Q_3$ 10.5 / 54 60 68  $\frac{Q_3 - 8.5}{2} = \frac{6}{14} \Rightarrow Q_3 = \underline{9.36}$

e) $Q_2 - Q_1 > Q_3 - Q_2$ $\therefore$ negative Skew
(2.31)  (1.44)



6. The time taken, in minutes, by children to complete a mathematical puzzle is assumed to be normally distributed with mean $\mu$ and standard deviation $\sigma$. The puzzle can be completed in less than 24 minutes by 80% of the children. For 5% of the children it takes more than 28 minutes to complete the puzzle.

(a) Show this information on the Normal curve below. **(2)**

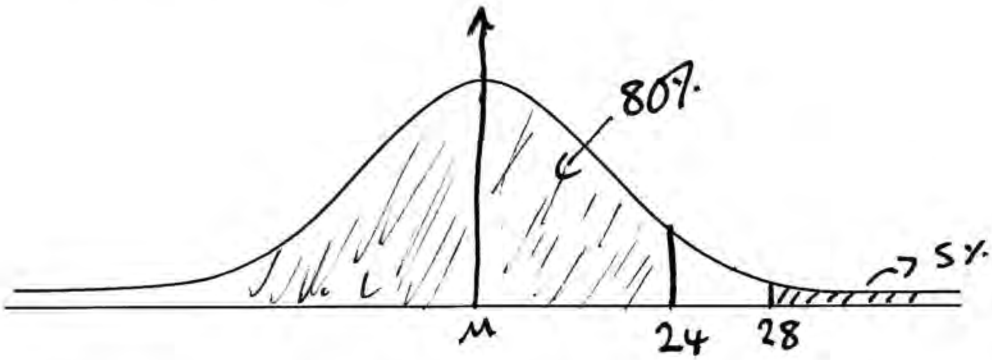
(b) Write down the percentage of children who take between 24 minutes and 28 minutes to complete the puzzle. **(1)**

(c) (i) Find two equations in $\mu$ and $\sigma$.

(ii) Hence find, to 3 significant figures, the value of $\mu$ and the value of $\sigma$. **(7)**

A child is selected at random.

(d) Find the probability that the child takes less than 12 minutes to complete the puzzle. **(3)**




b) 15% c) $P(t>28) = 5\% \Rightarrow P(z > \frac{28-\mu}{\sigma}) = 5\%$

$\therefore \frac{28-\mu}{\sigma} = 1.6449 \quad \therefore 28-\mu = 1.6449\sigma$

$P(t>24) = 20\% \Rightarrow P(z > \frac{24-\mu}{\sigma}) = 20\%$

$\frac{24-\mu}{\sigma} = 0.8416 \quad 24-\mu = 0.8416\sigma$

ii) $4 = 0.8033\sigma \quad \therefore \sigma = 4.979$

$\mu = 24 - 0.8416\sigma \quad \therefore \mu = 19.8$

$\therefore \mu = 19.8 \quad \sigma = 4.98$

d) $P(t<12) = P(z < \frac{12-19.8}{4.98})$

$\approx P(z < -1.57) = 1 - \Phi(1.57) = 0.0582$

**7.** In a large company,

78% of employees are car owners,
30% of these car owners are also bike owners,
85% of those who are not car owners are bike owners.

(a) Draw a tree diagram to represent this information.
**(3)**

An employee is selected at random.

(b) Find the probability that the employee is a car owner or a bike owner but not both.
**(2)**

Another employee is selected at random.

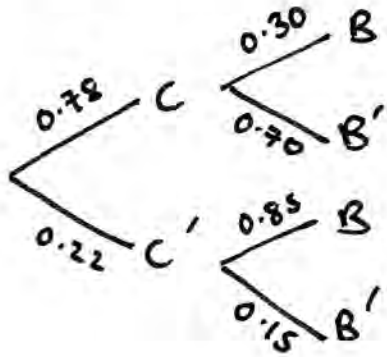
Given that this employee is a bike owner,

(c) find the probability that the employee is a car owner.
**(3)**

Two employees are selected at random.

(d) Find the probability that only one of them is a bike owner.
**(3)**

a)




b) $P(CB) + P(BC)$

$= 0.78 \times 0.70 + 0.22 \times 0.85$

$= 0.733$

c) $P(C|B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{0.78 \times 0.3}{P(B)}$

$P(B) = 0.78 \times 0.3 + 0.22 \times 0.85$

$= 0.421$

$\therefore\ P(C|B) = \dfrac{0.234}{0.421} = 0.56$

d) $BB' + B'B = 0.421 \times 0.579 \times 2$

$= 0.488$