•S1 JUNE 05

①

1) A = −0.79 evidence to suggest negative correlation as x increases, y decreases.

B = 0.08 no evidence of correlation, no pattern.

C = 0.68 evidence to suggest positive correlation, as x increases y increases.

2)

Distance	(cw)	freq	(cf)	freq density
41–45	(5)	4	(4)	0.8
46–50	(5)	19	(23)	3.8
51–60	(10)	53	(76)	5.3
61–70	(10)	37	(113)	3.7
71–90	(20)	15	(128)	0.75
91–150	(60)	6	(134)	0.1

b) $n = 134$ $\qquad b + \frac{(qn - cf)}{fc} \times cw$

Q_1 $\frac{1}{4}n = 33.5$ $\quad x_{34} = 51\text{–}60 \Rightarrow 50.5 + \left(\frac{33.5-23}{53}\right) \times 10$

Q_2 $\frac{1}{2}n = 67$ $\quad x_{67}/x_{68} = 51\text{–}60 \Rightarrow 50.5 + \left(\frac{67-23}{53}\right) \times 10$

Q_3 $\frac{3}{4}n = 100.5$ $\quad x_{101} = 61\text{–}70 \Rightarrow 60.5 + \left(\frac{100.5-76}{37}\right) \times 10$

$Q_1 = 52.48 \qquad Q_2 = 58.8 \qquad Q_3 = 67.12$

c) $\text{Mean} = \frac{\Sigma fx}{n} = \frac{8379.5}{134} = 62.53$

②

$s.d.^2 = \frac{\Sigma fx^2}{n} - \text{mean}^2 = \frac{557489.75}{134} - 62.53^2$

$s.d.^2 = (\text{var}) = 250.37 \Rightarrow s.d. = 15.82$

e) $\frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1} = \frac{67.12 - 2(58.8) + 52.48}{67.12 - 52.48}$

skew = 0.137 slight positive skew

$Q_2 - Q_1 = 6.32 \qquad Q_3 - Q_2 = 8.32$

$Q_2 - Q_1 < Q_3 - Q_2 \Rightarrow$ positive skew

3) $b = \frac{S_{xy}}{S_{xx}}$ $\qquad S_{xy} = \Sigma xy - \frac{(\Sigma x)^2}{n} = 8880 - \frac{(130)(48)}{8}$

$S_{xy} = 8100$

$b = \frac{8100}{20487.5}$

$b = 0.395 \qquad a = \bar{y} - b\bar{x} = \left(\frac{48}{8}\right) - 0.395\left(\frac{130}{8}\right) = -0.425$

$y = -0.425 + 0.395x$

b) $(f - 100) = -0.425 + 0.395(m - 250)$

$f = 99.575 + 0.395m - 98.75$

$f = 0.825 + 0.395m$

c) $f = 0.825 + 0.395(235) = 93.65$ lt

③

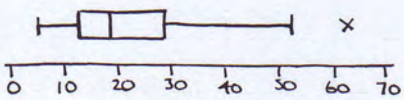
4) min = 5 max = 63 $Q_1 = 12$ $Q_2 = 17$ $Q_3 = 28$

lower limit $= Q_1 - 1.5 IQR = 12 - 1.5(28-12) = 12 - 24$
$= -12$

$\Rightarrow$ no outliers exist

upper limit $= Q_3 + 1.5 IQR = 28 + 24 = 52$

$\Rightarrow$ 63 is an outlier.



b) positive skew since $Q_2 - Q_1 < Q_3 - Q_2$
(5) (11)

c) majority of delays are fairly small.

5)

x	1	2	3	4	5
P	k	2k	3k	5k	6k
P	$\frac{1}{17}$	$\frac{2}{17}$	$\frac{3}{17}$	$\frac{5}{17}$	$\frac{6}{17}$

$\Sigma P = 1$
$\Rightarrow k + 2k + 3k + 5k + 6k = 1$
$17k = 1 \Rightarrow k = \frac{1}{17}$

$\downarrow x$

b) $E(x) = \frac{1}{17} + \frac{4}{17} + \frac{9}{17} + \frac{20}{17} + \frac{30}{17} = \frac{64}{17} = 3.76$

$E(x^2) = 1^2 \times \frac{1}{17} + 2^2 \times \frac{2}{17} + 3^2 \times \frac{3}{17} + 4^2 \times \frac{5}{17} + 5^2 \times \frac{6}{17}$

$E(x^2) = \frac{1}{17} + \frac{8}{17} + \frac{27}{17} + \frac{80}{17} + \frac{150}{17} = \frac{266}{17} = 15.64$

c) $V(x) = E(x^2) - E(x)^2 = 15.64 - 3.76^2 = 1.474$

(use exact answers)

d) $V(4 - 3x) = (-3)^2 V(x) = 9 \times 1.47 = 13.23$ $\quad = 13.2$ (1dp)

④

6) $M \sim N(155, 3.5^2)$

a) $P(M > 160) \Rightarrow P\left(Z > \frac{160-155}{3.5}\right) = P(Z > 1.43) = 1 - \Phi(1.43)$
$= 0.0764$

b) $P(150 < M < 157) \Rightarrow P\left(\frac{150-155}{3.5} < Z < \frac{157-155}{3.5}\right)$

$= P(-1.43 < Z < 0.57)$

$= \Phi(0.57) - \Phi(-1.43)$

$= \Phi(0.57) - (1 - \Phi(1.43))$

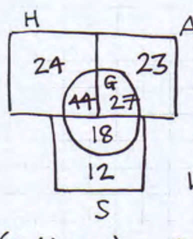
$= 0.7157 - 0.0764 = 0.6393$

c) $P(M < m) = 0.3 \Rightarrow P\left(Z < \frac{m-155}{3.5}\right) = \Phi\left(\frac{m-155}{3.5}\right) = 0.3$

$\Phi\left(\frac{155-m}{3.5}\right) = 0.7 = \Phi(0.52)$

$\frac{155-m}{3.5} = 0.52 \Rightarrow 155 - m = 1.82 \Rightarrow 153.18$ min

7)



$89 - 44 - 18 = 27$

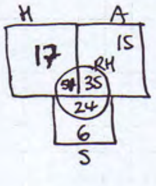
$148 - 89 - 24 - 12 = 23$

148

a) $P(\text{Art}) = \frac{50}{148} = \frac{25}{74}$

b) $P(G' | \text{Arts}) = \frac{23}{50}$

c)



S ⇒ 80% of 30 = 24

H ⇒ 75% of 68 = 51

A ⇒ 70% of 50 = 35

$P(RH) = \frac{110}{148} = \frac{55}{74}$

d) $P(S | RH) = \frac{P(S \cap RH)}{P(RH)}$

$= \frac{24}{\ldots} = \frac{12}{\ldots}$