①

S1 May 2002 - Solutions

1 a) $\frac{5}{6}$ b) $\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{25}{216}$ $(=0.116)$

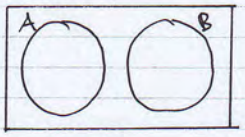
---

2. • A real world problem is observed and a mathematical model is devised.
• The model is used to make predictions and experimental data is collected
• Expected outcomes are obtained
• The model is refined in light of the actual outcomes.

---

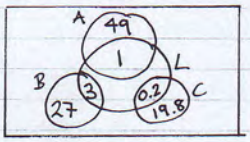
3. a) The probability of B given that A has occured.

b)



c) $\frac{1}{2}\times\frac{2}{100} = \frac{1}{100}$

d)



2% of 50% = 1%
10% of 30% = 3%
1% of 20% = 0.2%

e) $\frac{4.2}{100} = \frac{42}{1000} = \frac{21}{500}$ $(=0.042)$

---

②

4. a)

X	1	2	3	4	5	6	7	8
F(x)	0.1	0.2	0.25	0.4	0.5	0.6	0.75	1
P(X=x)	0.1	0.1	0.05	0.15	0.1	0.1	0.15	0.25

b) $E(X) = 1\times0.1 + 2\times0.1 + 3\times0.05 + 4\times0.15 + 5\times0.1 + 6\times0.1 + 7\times0.15 + 8\times0.25$
$= \underline{5.2}$

$Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = 1^2\times0.1 + 2^2\times0.1 + 3^2\times0.05 + 4^2\times0.15 + 5^2\times0.1 + 6^2\times0.1 + 7^2\times0.15 + 8^2\times0.25$
$= 32.8$

$Var(X) = 32.8 - 5.2^2 = \underline{5.76}$

c) $E(2X+3) = 2E(X)+3 = \underline{13.4}$

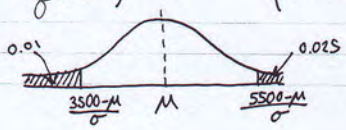
$Var(2X+3) = 2^2 Var(X) = \underline{23.04}$

---

5 a) • Bell shaped.
• Symmetrical.

b) $P(X\leq3500) = 0.01$ $P(X\geq5500) = 0.025$

$P\left(Z\leq\frac{3500-\mu}{\sigma}\right) = 0.01$ $P\left(Z\geq\frac{5500-\mu}{\sigma}\right) = 0.025$



---

③

$\therefore P\left(Z\leq\frac{\mu-3500}{\sigma}\right) = 0.99$

$\Rightarrow \frac{\mu-3500}{\sigma} = 2.32$ ①

$P\left(Z\leq\frac{5500-\mu}{\sigma}\right) = 0.975$

$\Rightarrow \frac{5500-\mu}{\sigma} = 1.96$ ②

from ① $\mu = 2.32\sigma + 3500$

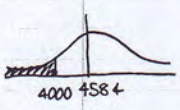
sub in ② $5500-(2.32\sigma+3500) = 1.96\sigma$

$2000 = 4.28\sigma$

$\sigma = \frac{2000}{4.28} = \underline{467}$

$\therefore \mu = 2.32\times\frac{2000}{4.28} + 3500 = \underline{4584}$

c) $P(X\leq4000) = P\left(Z\leq\frac{4000-4584}{467}\right)$



$= 1 - P\left(Z\leq\frac{4584-4000}{467}\right)$

$= 1 - \Phi(1.2505) = 1-0.89$

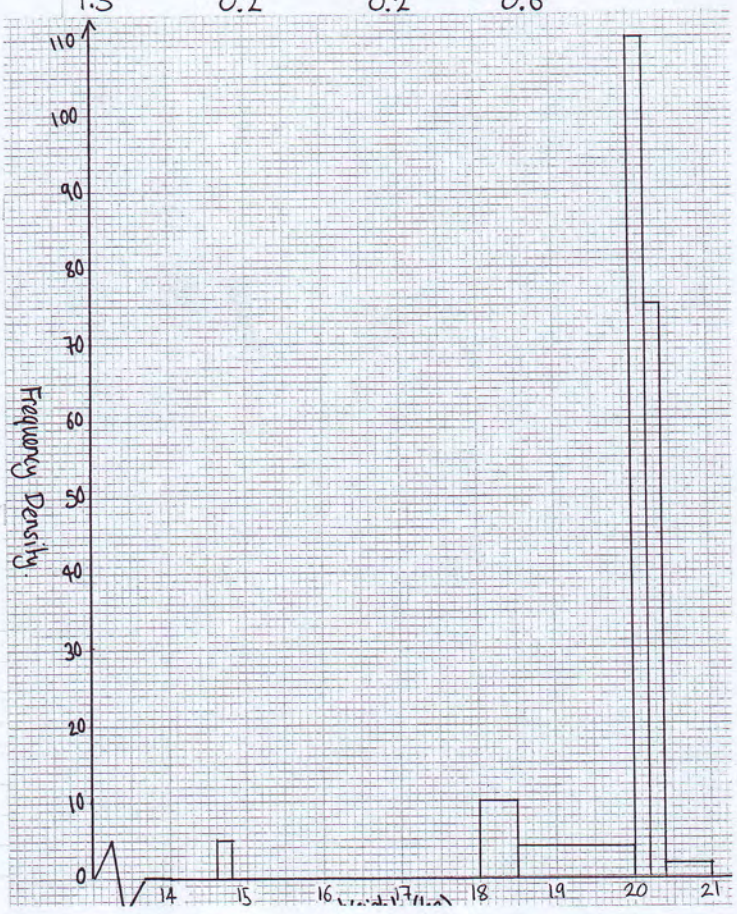
$= \underline{11\%}$

---

④

6. a) Frequency densities: $\frac{1}{0.2} = 5$, $\frac{0}{0.2} = 0$, $\frac{5}{0.5} = 10$

$\frac{6}{1.5} = 4$, $\frac{22}{0.2} = 110$, $\frac{15}{0.2} = 75$, $\frac{1}{0.6} = 1.7$



(5)

b)

Weight in kg	Frequency	x	y	fy
14.6 – 14.8	1	14.7	7	7
14.8 – 18.0	0	14.9	9	0
18.0 – 18.5	5	18.25	42.5	212.5
18.5 – 20.0	6	19.25	52.5	315
20.0 – 20.2	22	20.1	61	1342
20.2 – 20.4	15	20.3	463	945
20.4 – 21.0	1	20.7	67	67
				2888.5

$\mu_y = \frac{\Sigma fy}{\Sigma f} = \frac{2888.5}{50} = 57.77$

$\sigma_y = \sqrt{\frac{\Sigma fy^2}{\Sigma f} - \mu_y^2} = \sqrt{\frac{171503.75}{50} - 57.77^2}$

$= 9.6281.....$

$\therefore \mu_x = \frac{\mu_y}{10} + 14 = \underline{19.777}$

$\sigma_x = \frac{\sigma_y}{10} = \underline{0.963}$ (3sf)

c) $\frac{50+1}{2} = 25.5 \Rightarrow$ median is in class 20.0-20.2

$\therefore Q_2 = 20.0 + \left(\frac{\frac{50}{2} - 12}{22}\right) \times 0.2 = \underline{20.118}$

d) Median as the data is skewed.

---

(6)

7. a) $S_{cc} = \Sigma c^2 - \frac{(\Sigma c)^2}{n}$

$= 14245 - \frac{357^2}{10} = 1500.1$

$S_{tt} = \Sigma t^2 - \frac{(\Sigma t)^2}{n}$

$= 3025 - \frac{169^2}{10} = 168.9$

$S_{tc} = \Sigma tc - \frac{\Sigma t \Sigma c}{10}$

$= 6526 - \frac{169 \times 357}{10} = 492.7$

PMCC $= r = \frac{S_{tc}}{\sqrt{S_{cc} S_{tt}}} = \frac{492.7}{\sqrt{168.9 \times 1500.1}} = \underline{0.979}$ (3sf)

b) Yes as r is close to 1.

c) $c = a + bt$

$b = \frac{S_{tc}}{S_{tt}} = \frac{492.7}{168.9} = 2.92$ (3sf)

$a = \bar{c} - b\bar{t} = \frac{357}{10} - 2.92 \times \frac{169}{10} = -13.6$ (3sf)

$\underline{c = 2.92t - 13.6}$

d) For every 1°C rise in temp. approx. 3 more ice creams are sold.

---

(7)

e) $c = 2.92 \times 16 - 13.6 = 33.12$

so 33 icecreams

f) because it would be outside the range of data used to find the regression equation.