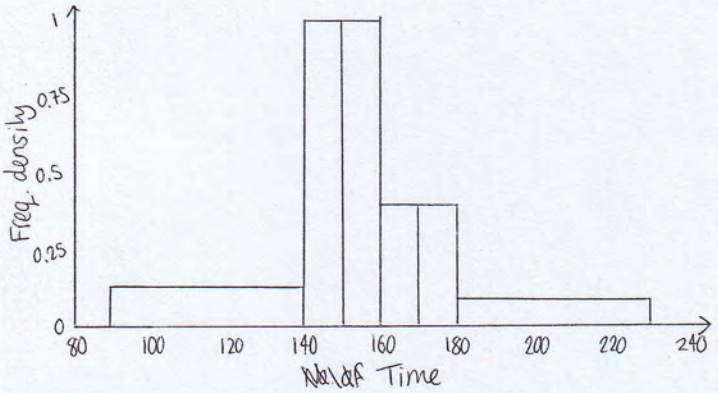
(1) S1 Solutions Jan 03

Q1

Time	No. of days	Class width	Freq density
90-139	8	50	$\frac{8}{50} = 0.16$
140-149	10	10	$\frac{10}{10} = 1$
150-159	10	10	$\frac{10}{10} = 1$
160-169	4	10	$\frac{4}{10} = 0.4$
170-179	4	10	$\frac{4}{10} = 0.4$
180-229	4	50	$\frac{4}{50} = 0.08$



(2) Q2 a) $P(A \cap B) = \frac{10}{100} = 0.1$

b) $P(A') = \frac{35+40}{100} = 0.75$

c) $P(B'/A) = \frac{P(A \cap B')}{P(A)} = \frac{15/100}{25/100} = \frac{15}{25} = 0.6$

d) $P(A \cap B) = 0.1$ $P(A) \times P(B) = 0.25 \times 0.5 = 0.125$

$\therefore$ not independent

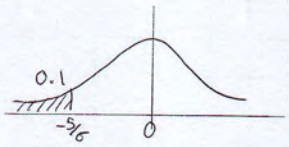
This implies that one make is less reliable than the other.

Q3 a) amount of coffe $= X$ $X \sim N(55, \sigma^2)$

$P(X < 50) = 0.1$

$P\left(\frac{X-55}{\sigma} < \frac{50-55}{\sigma}\right) = 0.1$

$P\left(Z < \frac{-5}{\sigma}\right) = 0.1$



$P\left(Z < \frac{5}{\sigma}\right) = 0.9$

$\Phi\left(\frac{5}{\sigma}\right) = 0.9$

$\frac{5}{\sigma} = 1.28$ $\sigma = \frac{5}{1.28} = 3.91\text{ml}$

(3) b) $X \sim N(\mu, 3^2)$

$P(X < 50) = 0.025$

$P\left(Z < \frac{50-\mu}{3}\right) = 0.025$

$P\left(Z < \frac{\mu-50}{3}\right) = 0.975$

$\Phi\left(\frac{\mu-50}{3}\right) = 0.975$

$\frac{\mu-50}{3} = 1.96$

$\mu = 55.88\text{ml}$

Q4 a) 14, 14, 14, 15, 15, 15, 15, 15, 15, 16, 16, 16, 16, 16, 16, 17, 17, 17, 18, 25

$\frac{20+1}{2} = 10.5 \Rightarrow$ median $= 16$

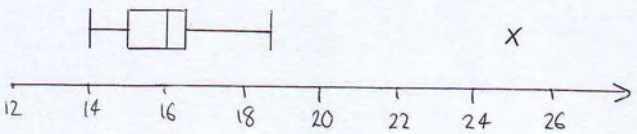
$\frac{20+1}{4} = 5.25 \Rightarrow Q1 = 15$

$3\left(\frac{20+1}{4}\right) = 15.75 \Rightarrow Q3 = \frac{16+17}{2} = 16.5$

b) $UOL = 1.5 \times (Q3-Q1) + Q3 = 18.75$

$LOL = Q1 - 1.5 \times (Q3-Q1) = 12.75$

(4)



c) mean $= \frac{322}{20} = 16.1$

d) There is a slight negative skew as

$Q_3 - Q_2 < Q_2 - Q_1$

OR Data is roughly symmetrical as mean $\approx$ median $(16 \approx 16.1)$

Q5 a)

x	0	1	2	3
$P(X=x)$	$2k$	k	0	k

$2k + k + 0 + k = 1$

$4k = 1$

$k = 0.25$

b) $E(X) = 0 \times 0.5 + 1 \times 0.25 + 2 \times 0 + 3 \times 0.25$

$= 0 + 0.25 + 0 + 0.75$

$= 1$

$E(X^2) = 0^2 \times 0.5 + 1^2 \times 0.25 + 2^2 \times 0 + 3^2 \times 0.25$

$= 0.25 + 2.25 = 2.5$

(5) c) $\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 2.5 - 1^2 = \underline{1.5}$

$\text{Var}(3X-2) = 3^2\text{Var}(X)$

$= 9 \times 1.5 = \underline{13.5}$

d) $P(X_1+X_2=5) = P(X_1=3 \cap X_2=2)$

$+ P(X_1=2 \cap X_2=3)$

$= 0+0 = \underline{0}$

e) $Y = X_1+X_2$

y	$P(Y=y)$
0	$P(X_1=0) \times P(X_2=0) = \underline{0.25}$
1	$P(X_1=1) \times P(X_2=0) + P(X_2=1) \times P(X_1=0) = \underline{0.25}$
2	0.0625
3	0.25
4	0.125
5	0
6	0.0625

f) $P(1.3 < X_1+X_2 \leq 3.2) = P(Y=2)+P(Y=3)$

$= 0.0625 + 0.25 = \underline{0.3125}$

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(6) Q6 a)

Year	x	y
1990	20	24
1991	26	38
1992	32	42
1993	34	44
1994	37	43
1995	44	52
1996	48	59
1997	50	66
1998	53	70
1999	58	79

x is the explanat variable as the numb of new cars sold is dependent on the cost of the advertising.

b) $y = a+bx$

$b = \frac{S_{xy}}{S_{xx}}$ $S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 22611 - \frac{402 \times 517}{10}$

$= \underline{1827.6}$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 17538 - \frac{402^2}{10}$

$= \underline{1377.6}$

$\therefore b = \frac{1827.6}{1377.6} = 1.326655\ldots$

$a = \bar{y} - b\bar{x} = \frac{\Sigma y}{n} - b\frac{\Sigma x}{n} = \frac{517}{10} - 1.3266 \times \frac{402}{10}$

$= -1.631533\ldots$

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(7)

$\therefore \underline{y = 1.33x - 1.63}$ (3sf)

c) $\frac{1}{10}(c-4000) = 1.33(p-100) - 1.63$

$c-4000 = 13.3(p-100) - 16.3$

$c = 13.3p + 2653.7$

$\underline{c = 13.3p + 2650}$ (3sf)

d) If no money is spent on advertising you would sell 2650 cars.

e) 13.3 × 2 = 26.6 extra cars

Only valid for period 1990-1999.