Statistics 1 Jan 02 - Solutions

1 a) I) A statistical experiment tests if a mathematical model well represents a real life situation.

II) An event is where a there is a probability of several outcomes related to the event e.g rolling a die, tossing a coin.

b) Advantage - you can predict the outcomes of complicated events.
Disadvantage - it cannot possibly take all factors into account.

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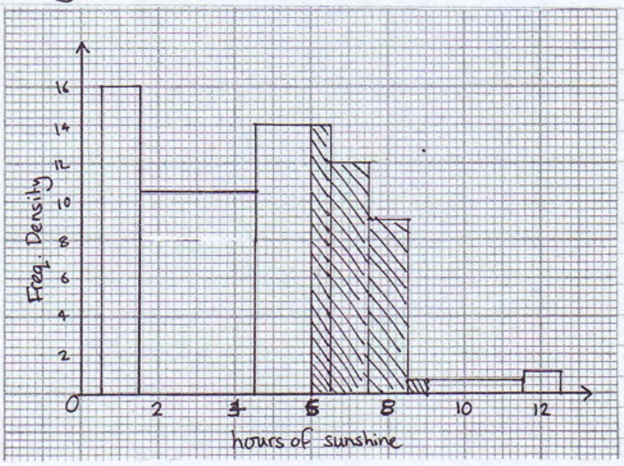
2. a) Frequency densities: $\frac{16}{1} = 16$, $\frac{32}{3} = 10\frac{2}{3}$, $\frac{28}{2} = 14$, 12

9, $\frac{2}{3}$, 1



b) $0.5 \times 14 + 12 + 9 + 0.5 \times \frac{2}{3} = \underline{28\frac{1}{3}}$

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3 a) $E(X) = 0 \times \frac{1}{3} + 1 \times a + 2\left(\frac{2}{3} - a\right) = \frac{5}{6}$

$a + \frac{4}{3} - 2a = \frac{5}{6}$

$a = \frac{4}{3} - \frac{5}{6}$

$\underline{a = \frac{1}{2}}$

b) $Var(X) = E(X^2) - [E(X)]^2$

$= 0^2 \times \frac{1}{3} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{6} - \left(\frac{5}{6}\right)^2$

$= \frac{1}{2} + \frac{4}{6} - \frac{25}{36}$

$= \underline{\frac{17}{36}}$

c) $P(X \leq 15) = \underline{1}$

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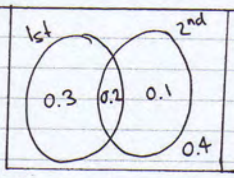
4. 

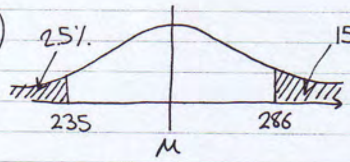
a) $\underline{0.4}$

b) $0.3 + 0.1 = \underline{0.4}$

c) $P(W_2 | W_1') = \frac{P(W_2 \cap W_1')}{P(W_1')} = \frac{0.1}{0.5} = \underline{0.2}$

d) $P(W_1 \cap W_2) = 0.2 \quad P(W_1) \times P(W_2) = 0.5 \times 0.3 = 0.15$

$\therefore$ not independent.

5. a) 

$X \sim N(\mu, \sigma^2) \qquad Z = \frac{X - \mu}{\sigma} \sim N(0, 1^2)$

$P(X < 235) = P\left(Z < \frac{235 - \mu}{\sigma}\right) = 0.025$

$= \Phi\left(\frac{235 - \mu}{\sigma}\right) = 0.025$

$\frac{235 - \mu}{\sigma} = -1.96$

$\underline{\mu - 235 = 1.96\sigma}$ (1)

b) $P\left(Z > \frac{286 - \mu}{\sigma}\right) = 0.15$

$\frac{286 - \mu}{\sigma} = 1.0364$

$\underline{286 - \mu = 1.0364\sigma}$ (2)

c) (1) + (2) $\quad 286 - 235 = (1.96 + 1.0364)\sigma$

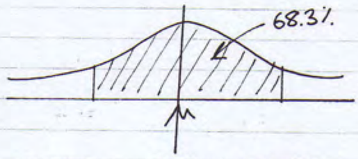
$51 = 2.9964\sigma$

$\underline{\sigma = 17.0}$ (3sf)

$\therefore$ sub in (1) $\quad \mu - 235 = 1.96 \times 17.0$

$\mu = 33.36 + 235$

$\underline{\mu = 268}$ (3sf)

d) 

$\mu \pm \sigma = 268 \pm 17.0$

$= \underline{251 \text{ and } 285}$ days

(5)

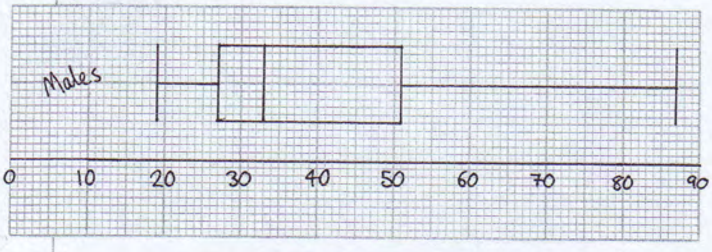
6. a) $Q_1 : \frac{15}{4} = 3.75 \therefore 4^{th}\text{value} = \underline{\underline{27}}$

$Q_2 : \frac{15}{2} = 7.5 \therefore 8^{th}\text{value} = \underline{\underline{33}}$

$Q_3 : \frac{15 \times 3}{4} = 11.25 \therefore 12^{th}\text{value} = \underline{\underline{51}}$

$\therefore IQR = 51 - 27 = \underline{24}$.

b)



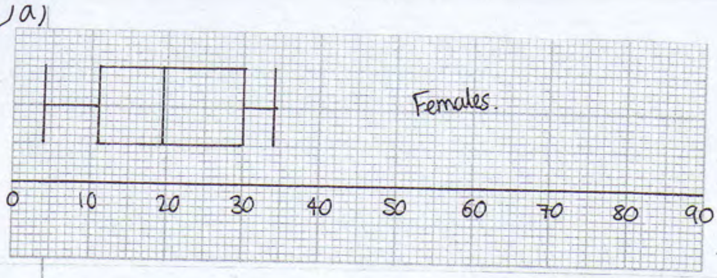
c) $\mu = \frac{19+21+26+27+27+32+32+33+34+38+45+51+60+86+87}{15}$

$= \frac{618}{15} = 41.2$

$\sigma^2 = \frac{\sum x^2}{n} - \mu^2 = \frac{31864}{15} - 39.2^2 = 587.6$

$\sigma = \sqrt{587.6} = \underline{\underline{24.2}}$ (3sf)

(6) a)



e) • Males have a positive skew, Females have a very small positive skew
• Range for males > Range Females.

7. a) PTO.

b) $r = \frac{S_{ts}}{\sqrt{S_{tt} S_{ss}}}$ $\quad S_{ts} = \sum ts - \frac{\sum t \sum s}{n}$

$= 396775 - \frac{1340 \times 2310}{10}$

$= 87235$

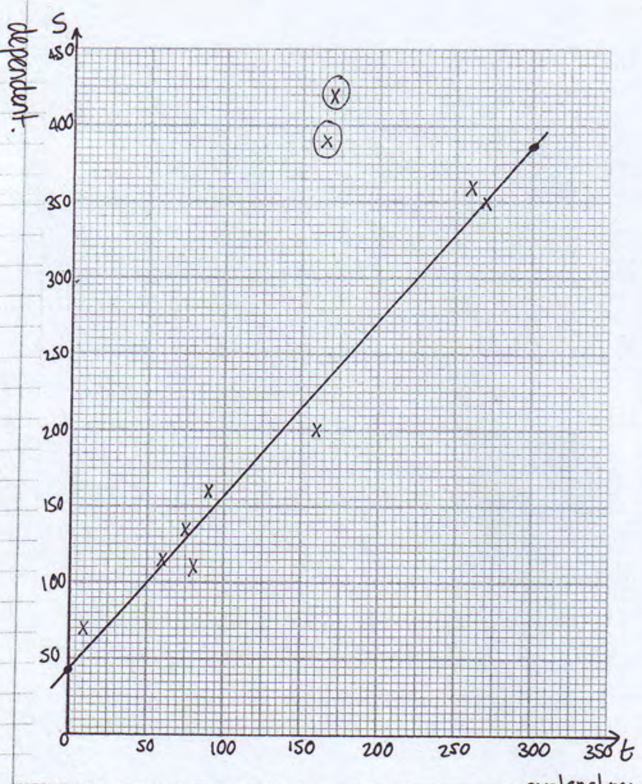
$S_{tt} = \sum t^2 - \frac{(\sum t)^2}{n}$

$= 246050 - \frac{1340^2}{10} = 66490$

$S_{ss} = 694650 - \frac{2310^2}{10} = 161040$

$\therefore r = \frac{87235}{\sqrt{66490 \times 161040}} = \underline{\underline{0.843}}$ (3sf)

(7)



c) The PMCC will not change as adding will not effect the spread of values.

d) $s = a + bt \quad b = \frac{S_{st}}{S_{tt}} = \frac{72587}{63671.875} = 1.14$ (3sf)

$a = \bar{s} - b\bar{t} = 187.5 - 1.14 \times 125.625$
$= 44.3$ (3sf)

$\underline{s = 44.3 + 1.14t}$

(8)

f) • the gradient of the line would be increased.
• any predictions from the line would be less accurate.