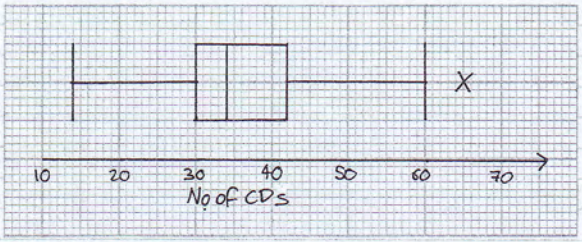
S1 Jan'01 Solutions

1. $Q_1 = 30,\ Q_2 = 34,\ Q_3 = 42$

$1.5(Q_3 - Q_1) = 1.5 \times (42-30) = 18$

$\therefore$ UOL $= 42 + 18 = 60$
LOL $= 30 - 18 = 12$

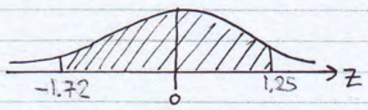


2. a) $X \sim N(177, 6.4^2)$

$Z\ \dfrac{X-177}{6.4} \sim N(0, 1^2)$

$P(166 < X < 185) = P\left(\dfrac{166-177}{6.4} < Z < \dfrac{185-177}{6.4}\right)$

$= P(-1.72 < Z < 1.25)$



$= \Phi(1.25) - \Phi(-1.72)$

$= 0.8944 - (1 - 0.9573) = \underline{0.8517}$

b) • Most male heights are clustered about 177cm.
• Hight is a continuous random variable.

c) • Simplifies a real world problem.
• Gives us a quick/cheap solution.

3. a) $P(Y=y) = \frac{1}{6}$ $\quad y = 1,2,3,4,5,6$

b) Discrete uniform distribution.

c) $E(6Y+2) = 6E(Y) + 2$

$= 6\left(\dfrac{6+1}{2}\right) + 2 = \underline{\underline{23}}$

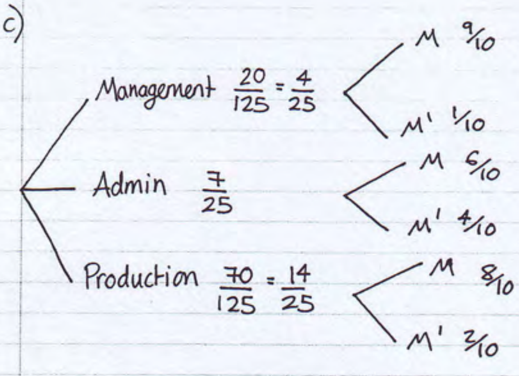
d) $Var(4Y-2) = 4Var(Y)$

$= 4^2\left(\dfrac{(6+1)(6-1)}{12}\right) = \cancel{11.7}\ \underline{\underline{46.7}}$

4. a) $\dfrac{35}{125} = \underline{\underline{\dfrac{7}{25}}}$

b) $\dfrac{6}{20} = \underline{\underline{\dfrac{3}{10}}}$

c)



d) $P(M) = \dfrac{4}{25} \times \dfrac{9}{10} + \dfrac{7}{25} \times \dfrac{6}{10} + \dfrac{14}{25} \times \dfrac{8}{10}$

$= \dfrac{190}{250} = \underline{\underline{\dfrac{19}{25}}}$

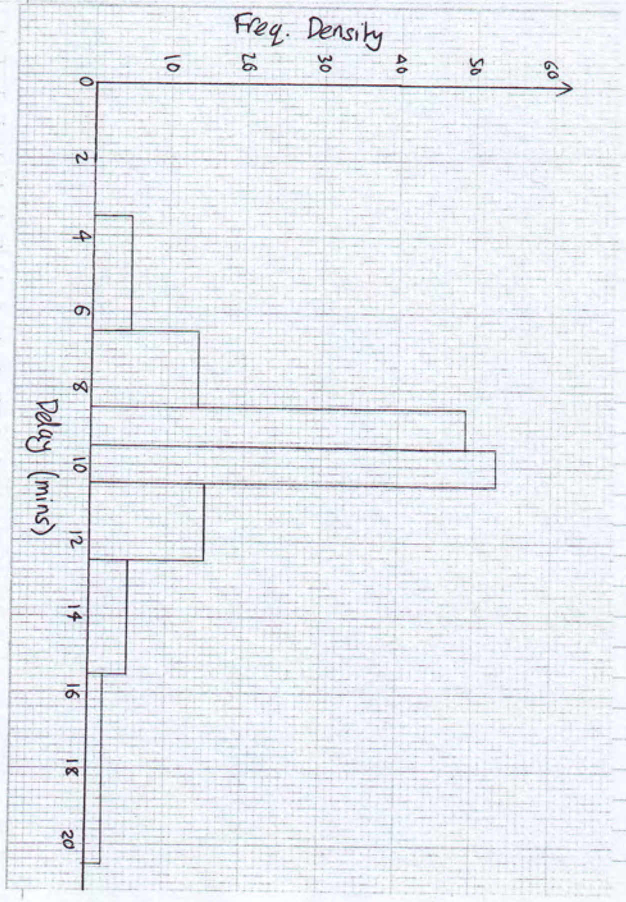
e) $P(\text{Production} \mid M) = \dfrac{P(M \cap \text{Production})}{P(M)}$

$= \dfrac{\frac{14}{25} \times \frac{8}{10}}{19/25}$

$= \underline{\underline{\dfrac{56}{95}}}$

5. a) Frequency Densities

$\dfrac{15}{3} = 5,\ \dfrac{28}{2} = 14,\ 49,\ 53,\ \dfrac{30}{2} = 15,\ \dfrac{15}{3} = 5,\ \dfrac{10}{5} =$



b) Data is continuous

c) Median is in the 10min class

$$Q_2 = 9.5 + \frac{\left(\frac{200}{2} - 92\right)}{53} \times 1$$

$$= 9.65$$

d)

Delay (mins)	F	x	fx	fx^2
4-6	15	5	75	375
7-8	28	7.5	210	1575
9	49	9	441	3969
10	53	10	530	5300
11-12	30	11.5	345	3967.5
13-15	15	14	210	2940
16-20	10	18	180	3240
	200		1991	21366.5

$$\mu = \frac{\Sigma fx}{\Sigma f} = \frac{1991}{200} = 9.955$$

$$\sigma^2 = \frac{\Sigma fx^2}{\Sigma f} - \mu^2 = \frac{21366.5}{200} - 9.955^2$$

$$= 7.730$$

$$\sigma = \sqrt{7.730} = 2.780$$

e) $\frac{3(\mu - Q_2)}{\sigma} = \frac{3 \times (9.955 - 9.65)}{2.780} = 0.329$ (3d

F) The Normal dist. requires the data to be symmetric about the mean. This data shows a positive skew

6. a) $S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 65.68 - \frac{(25)^2}{10}$

$= 3.18$

$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 130.64 - \frac{25 \times 50}{10}$

$= 5.64$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 260.48 - \frac{50^2}{10}$

$= 10.48$

b) $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{5.64}{\sqrt{3.18 \times 10.48}} = 0.977$

c) It shows a very strong positive correlation

d) $a = \bar{y} - b\bar{x}$ $\quad b = \frac{S_{xy}}{S_{xx}} = \frac{5.64}{3.18} = 1.774$

$a = \frac{\Sigma y}{n} - 1.774 \frac{\Sigma x}{n} = 5 - 1.774 \times 2.5 = 0.56$

e) Reconditioning a new machine $(x=0)$ costs £566.

f) I) 2400hrs $(x = 2.4)$

$y = 0.566 + 1.774x$

$y = 0.566 + 1.774 \times 2.4 = 4.824$

$\therefore$ cost = £4824

II) increase of 1500hrs (increase in x of 1.5)

$1.5 \times 1.774 = 2.661$

increase of £2661

g) because 4500hrs is outside the sample used.