

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6683/01)

**January 2008
6683 Statistics S1
Mark Scheme**

Question Number	Scheme	Marks
1. (a)	$\sum x = 773, \sum y = 724$ $r = \frac{10 \times 56076 - 773 \times 724}{\sqrt{(10 \times 60475 - 773^2)(10 \times 53122 - 724^2)}} \quad \text{o.e.}$ $r = 0.155357 \dots$	B1, B1 M1 A1ft A1
(b)	<p>Both weak correlation Neither score is a good indication of future performance Interview test is slightly better since correlation is positive</p>	(5) B1g B1h (2) Total 7 marks
NB (a)	$S_{xx} = 60475 - \frac{(773)^2}{10} = 722.1, \quad S_{yy} = 53122 - \frac{(724)^2}{10} = 704.4, \quad S_{xy} = 56076 - \frac{773 \times 724}{10} = 110.8$ <p>1st B1 for $\sum x$ and 2nd B1 for $\sum y$, should be seen or implied.</p> <p>M1 for at least one correct attempt at one of S_{xx}, S_{yy} or S_{xy} and then using in the correct formula</p> <p>1st A1ft for a fully correct expression. (ft their $\sum x$ and their $\sum y$) or 3 correct expressions for S_{xx}, S_{xy}, and S_{yy} but possibly incorrect values for these placed correctly in r.</p>	
(b)	<p>2nd A1 for awrt 0.155</p> <p>If $r > 0.5$ they can score B1g in (b) for saying that it (skills test) is not a good guide to performance but B0h since a second acceptable comment about both tests is not possible.</p> <p>Give B1 for one correct line, B1B1 for any 2. If the only comment is the test(s) <u>are</u> a good guide: scores B0B0 If the only comment is the tests are not good: scores B1B0 (second line)</p> <p>The third line is for a comment that suggests that the interview test is OK but the skills test is not since one is positive and the other is negative.</p> <p>Treat 1st B1 as B1g and 2nd as B1h</p> <p>An answer of “no” alone scores B0B0</p>	

Question Number	Scheme	Marks
<p>2.</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>mean is $\frac{2757}{12}, = 229.75$</p> <p>sd is $\sqrt{\frac{724961}{12} - (229.75)^2}, = 87.34045$</p> <p>[Accept $s =$ AWRT 91.2]</p> <p>Ordered list is: 125, 160, 169, 171, 175, 186, 210, 243, 250, 258, 390, 420</p> <p>$Q_2 = \frac{1}{2}(186 + 210) = 198$</p> <p>$Q_1 = \frac{1}{2}(169 + 171) = 170$</p> <p>$Q_3 = \frac{1}{2}(250 + 258) = 254$</p> <p>$Q_3 + 1.5(Q_3 - Q_1) = 254 + 1.5(254 - 170), = 380$ Accept AWRT (370-392)</p> <p>Patients F (420) and B (390) are outliers.</p> <p>$\frac{Q_1 - 2Q_2 + Q_3}{Q_3 - Q_1} = \frac{170 - 2 \times 198 + 254}{254 - 170}, = 0.\dot{3}$</p> <p>Positive skew.</p>	<p>AWRT 230 M1, A1</p> <p>AWRT 87.3 M1, A1</p> <p>(4)</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>M1, A1</p> <p>B1ft B1ft</p> <p>(4)</p> <p>AWRT 0.33 M1, A1</p> <p>A1ft</p> <p>(3)</p> <p>Total 14 marks</p>
<p>(a)</p> <p>NB</p> <p>(b)</p> <p>S.C.</p> <p>(c)</p> <p>(d)</p>	<p>1st M1 for using $\frac{\sum x}{n}$ with a credible numerator and $n = 12$.</p> <p>2nd M1 for using a correct formula, root required but can ft their mean</p> <p>Use of $s = \sqrt{8321.84...} = 91.22...$ is OK for M1A1 here.</p> <p>Answers only from a calculator in (a) can score full marks</p> <p>1st B1 for median= 198 only, 2nd B1 for lower quartile 3rd B1 for upper quartile</p> <p>If all Q_1 and Q_3 are incorrect but an ordered list (with ≥ 6 correctly placed) is seen and used then award B0B1 as a special case for these last two marks.</p> <p>M1 for a clear attempt using their quartiles in given formula,</p> <p>A1 for any value in the range 370 - 392</p> <p>1st B1ft for any one correct decision about B or F - ft their limit in range (258, 420)</p> <p>2nd B1ft for correct decision about both F and B - ft their limit in range (258, 420)</p> <p>If more points are given score B0 here for the second B mark. (Can score M0A0B1B1 here)</p> <p>M1 for an attempt to use their figures in the correct formula – must be seen (≥ 2 correct substitutions)</p> <p>1st A1 for AWRT 0.33</p> <p>2nd A1ft for positive skew. Follow through their value/sign of skewness . Ignore any further calculations. “positive correlation” scores A0</p>	

<p>3.</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Width</td> <td>1</td> <td>1</td> <td>4</td> <td>2</td> <td>3</td> <td>5</td> <td>3</td> <td>12</td> </tr> <tr> <td>Freq. Density</td> <td>6</td> <td>7</td> <td>2</td> <td>6</td> <td>5.5</td> <td>2</td> <td>1.5</td> <td>0.5</td> </tr> </table> <p style="text-align: right; margin-right: 20px;">0.5×12 or 6</p> <p>Total area is $(1 \times 6) + (1 \times 7) + (4 \times 2) + \dots = 70$</p> <p>$(90.5 - 78.5) \times \frac{1}{2} \times \frac{140}{\text{their } 70}$</p> <p style="text-align: right; margin-right: 20px;">“70 seen anywhere”</p> <p>Number of runners is 12</p>	Width	1	1	4	2	3	5	3	12	Freq. Density	6	7	2	6	5.5	2	1.5	0.5	<p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p style="text-align: right;">(5)</p> <p>Total 5 marks</p>
Width	1	1	4	2	3	5	3	12												
Freq. Density	6	7	2	6	5.5	2	1.5	0.5												
	<p>1st M1 for attempt at width of the correct bar (90.5 - 78.5) [Maybe on histogram or in table]</p> <p>1st A1 for 0.5×12 or 6 (may be seen on the histogram. Must be related to the area of the bar above 78.5 - 90.5.</p> <p>2nd M1 for attempting area of correct bar $\times \frac{140}{\text{their } 70}$</p> <p>B1 for 70 seen anywhere in their working</p> <p>2nd A1 for correct answer of 12.</p> <p>Minimum working required is $2 \times 0.5 \times 12$ where the 2 should come from $\frac{140}{70}$</p> <p>Beware $90.5 - 78.5 = 12$ (this scores M1A0M0B0A0)</p> <p>Common answer is $0.5 \times 12 = 6$ (this scores M1A1M0B0A0)</p> <p>If unsure send to review e.g. $2 \times 0.5 \times 12 = 12$ without 70 being seen</p>																			

<p>4.</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$S_{xy} = 1818.5 - \frac{41 \times 406}{10}, = 153.9$ <p>(could be seen in (b))</p> $S_{xx} = 188 - \frac{41^2}{10} = 19.9$ <p>(could be seen in (b))</p> $b = \frac{153.9}{19.9}, = 7.733668....$ $a = 40.6 - b \times 4.1 (= 8.89796....)$ $y = 8.89 + 7.73x$ <p>A typical car will travel 7700 miles every year</p> $x = 5, y = 8.89 + 7.73 \times 5 (= 47.5 - 47.6)$ <p>So mileage predicted is</p> <p style="text-align: right;">AWRT 48000</p>	<p>AWRT 154</p> <p>AWRT 7.73</p> <p>AWRT 48000</p> <p>M1, A1</p> <p>A1</p> <p>M1, A1</p> <p>M1</p> <p>A1</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>(4)</p> <p>(1)</p> <p>(2)</p> <p>Total 10 marks</p>
	<p style="text-align: center;">Accept calculations for S_{xx} and S_{xy} in (a) or (b)</p> <p>(a) M1 for correct attempt or expression for either 1st A1 for one correct 2nd A1 for both correct</p> <p>(b) Ignore the open marks for part (b) they should be awarded as per this scheme</p> <p>1st M1 for $\frac{\text{their } S_{xy}}{\text{their } S_{xx}}$ 1st A1 for AWRT 7.73 2nd M1 for attempt at correct formula for a (minus required). Ft their b. Quoting a correct formula but making one slip in sub.eg. $\bar{y} = 406$ is OK 2nd A1 for correct equation with 2dp accuracy. Accept $a = 8.89$, and $b = 7.73$ even if not written as final equation.</p> <p>Correct answers only (from calc) score 4/4 if correct to 2dp or 3/4 if AWRT 2dp</p> <p>(c) B1ft for their $b \times 1000$ to at least 2 sf. Accept “7.7 thousand” but value is needed</p> <p>(d) M1 for substituting $x = 5$ into their final answer to (b). A1 for AWRT 48000 (Accept “48 thousands”)</p>	

<p>5. (a)</p>	<p>Diagram may be drawn with $B \subset (A \cup C)$ or with the 0 for $B \cap (A \cup C)$' simply left blank</p> <div style="text-align: center;"> </div> <p>Accept decimals or probs. in Venn diagram</p> <p style="text-align: right;">3cc 90,3,2,1 1,(0),2 1 outside Box</p>	<p>M1 A1 M1A1 A1 B1</p> <p style="text-align: right;">(6)</p>
<p>(b)</p>	<p>$P(\text{none})=0.01$</p>	<p>B1ft</p>
<p>(c)</p>	<p>$P(A \text{ but not } B)=0.04$</p>	<p>M1 A1ft</p>
<p>(d)</p>	<p>$P(\text{any wine but } C)=0.03$</p>	<p>M1A1ft</p>
<p>(e)</p>	<p>$P(\text{exactly two})=0.06$</p>	<p>M1A1ft</p>
<p>(f)</p>	<p>$P(C A) = \frac{P(C \cap A)}{P(A)} = \frac{93}{96}$ or $\frac{31}{32}$ or AWRT 0.969</p>	<p>M1A1ft,A1</p>
<p>Total 16 marks</p>		
<p>(a)</p>	<p>1st M1 for 3 closed, labelled curves that overlap. A1 for the 90, 3, 2 and 1 2nd M1 for one of 1, 0 or 2 correct <u>or</u> a correct sum of 4 values for A, B or C 2nd A1 for all 7 values correct. Accept a blank instead of 0. NB final mark is a B1 for the box not an A mark as on EPEN In parts (b) to (f) full marks can be scored for correct answers or correct ft</p>	<div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>For M marks in (c) to (e) they must have a fraction</p> </div>
<p>(b)</p>	<p>B1ft Follow through their '1' from outside divided by 100</p>	
<p>(c)</p>	<p>M1 for correct expression eg $P(A \cup B) - P(B)$ or calculation e.g. 3 + 1 or 4 on top A1 for a correct probability, follow through with their '3+1' from diagram</p>	
<p>(d)</p>	<p>M1 for correct expression or calculation e.g. 1+2+0 or 99-96 or 3 on top A1 for a correct probability, follow through their '2+1+0' from diagram</p>	
<p>(e)</p>	<p>M1 for a correct expression or calculation e.g. 3+2+1 or 6 on top</p>	
<p>(f)</p>	<p>M1 for a correct expression upto " , " and <u>some</u> correct substitution, ft their values. One of these probabilities must be correct or correct ft. If P(C) on bottom M0 follow through their $A \cap C$ and their A but the ratio must be in (0, 1) 1st A1ft for correct answer only. Answer only scores 3/3, but check working $P(A \cap C) / P(C)$ is M0 2nd A1</p>	

<p>6. (a) (b) (c)</p>	<p>200 or 200g</p> <p>$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$ or $P(X > 210) = 0.2$ or diagram (o.e.) Correct use of 0.8 or 0.2</p> $Z = (\pm) \frac{210 - 200}{\sigma}$ $\frac{10}{\sigma} = 0.8416$ $\sigma = 11.882129\dots$ <p>0.8416 AWRT 11.9</p> $P(X < 180) = P\left(Z < \frac{180 - 200}{\sigma}\right)$ $= P(Z < -1.6832)$ $= 1 - 0.9535$ $= 0.0465 \text{ or AWRT } 0.046$	<p>B1</p> <p>M1 A1 M1 B1 A1</p> <p>M1 M1 A1</p> <p>(1) (5) (3)</p> <p>Total 9 marks</p>
<p>(a) (b) (c)</p>	<p>“mean = 200g” is B0 but “median = 200” or just “200” alone is B1</p> <p>Standardization in (b) and (c). They must use σ not σ^2 or $\sqrt{\sigma}$.</p> <p>1st M1 for a correct probability statement (as given or eg $P(200 < X < 210) = 0.3$ o.e.) or shaded diagram - must have values on z-axis and probability areas shown 1st A1 for correct use of 0.8 or $p = 0.2$. Need a correct probability statement. May be implied by a suitable value for z seen (e.g. $z = 0.84$) 2nd M1 for attempting to standardise. Values for x and μ used in formula. Don't need $z =$ for this M1 nor a z-value, just mark standardization. B1 for $z = 0.8416$ (or better) [$z = 0.84$ usually just loses this mark in (a)] 2nd A1 for AWRT 11.9</p> <p>1st M1 for attempting to Standardise with 200 and their sd(>0) e.g. $(\pm) \frac{180 - 200}{\text{their } \sigma}$ 2nd M1 NB on open this is an A mark ignore and treat it as 2nd M1 for 1 – a probability from tables provided compatible with their probability statement. A1 for 0.0465 or AWRT 0.046 (Dependent on both Ms in part (c))</p>	

7.(a)	$P(R = 3 \cap B = 0) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$	M1, A1 (2)																														
(b)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td>3</td><td>0</td><td>3</td><td>6</td><td>9</td></tr> <tr><td>2</td><td>0</td><td>2</td><td>4</td><td>6</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td><i>B</i></td><td></td><td></td><td></td><td></td></tr> <tr><td><i>R</i></td><td>0</td><td>1</td><td>2</td><td>3</td></tr> </tbody> </table>	3	0	3	6	9	2	0	2	4	6	1	0	1	2	3	0	0	0	0	0	<i>B</i>					<i>R</i>	0	1	2	3	All 0s All 1,2,3s All 4,6,9s B1 B1 B1 (3)
3	0	3	6	9																												
2	0	2	4	6																												
1	0	1	2	3																												
0	0	0	0	0																												
<i>B</i>																																
<i>R</i>	0	1	2	3																												
(c)	$a = \frac{7}{16}, b = c = d = \frac{1}{16}$	B1, B1 B1 (3)																														
(d)	$E(T) = \left(1 \times \frac{1}{16}\right) + \left(2 \times \frac{1}{8}\right) + \left(3 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{16}\right) + \dots$ $= 2\frac{1}{4} \text{ or exact equivalent e.g. } 2.25, \frac{9}{4}$	M1 A1 (2)																														
(e)	$\text{Var}(T) = \left(1^2 \times \frac{1}{16}\right) + \left(2^2 \times \frac{1}{8}\right) + \left(3^2 \times \frac{1}{8}\right) + \left(4^2 \times \frac{1}{16}\right) + \dots - \left(\frac{9}{4}\right)^2$ $= \frac{49}{4} - \frac{81}{16} = 7\frac{3}{16} \text{ or } \frac{115}{16} \quad (\text{o.e.})$	M1A1, M1 AWRT 7.19 A1 (4)																														
Total 14 marks																																
(a)	M1 for $\frac{1}{4} \times \frac{1}{4}$																															
(c)	1st B1 for $\frac{7}{16}$,																															
	2nd B1 for only one error in b, c, d ($b = c = d \neq \frac{1}{16}$ or $b = c = \frac{1}{16} \neq d$ etc), 3rd B1 all of $b, c, d = \frac{1}{16}$																															
(d)	M1 for attempting $\sum tP(T = t)$, 3 or more terms correct or correct ft. Must Attempt to sum.																															
	NB calculating $E(T)$ and then dividing by a number other than 1 scores M0 .																															
(e)	1st M1 for attempt at $E(T^2)$, 3 or more terms correct or correct ft.																															
	1st A1 for $\frac{49}{4}$ (o.e.) or a fully correct expression (all non-zero terms must be seen)																															
	2nd M1 for subtracting their $[E(T)]^2$, Must be some attempt to square $-\frac{9}{4}$ is M0 but $-\frac{9}{16}$ could be M1																															
	2nd A1 for correct fraction or AWRT 7.19 Full marks can still be scored in (d) and (e) if a is incorrect																															