

STATISTICS (C) UNIT 1 TEST PAPER 7

1. Twelve observations are made of a random variable X . This set of observations has mean 13 and variance $10\cdot2$.

Another twelve observations of X are such that $\Sigma x = 164$ and $\Sigma x^2 = 2372$.

Find the mean and the variance for all twenty-four observations. [6]

2. Given that $P(A) = \frac{3}{5}$, $P(B) = \frac{5}{8}$, $P(C) = \frac{1}{10}$, $P(A \text{ and } B) = \frac{7}{20}$, $P(A \text{ or } C) = \frac{7}{10}$, determine, with explanation, whether or not

(i) A and B are independent events, [2]

(ii) A and C are mutually exclusive events. [2]

(iii) Find the probability that **both** events A and C occur. [1]

3. The letters of the word GEOMETRIC are to be arranged in order.

(i) Find the number of distinct ways in which this can be done if there are no restrictions. [2]

(ii) Show that the number of distinct arrangements in which the vowels all come together is 8640. [3]

The letters of GEOMETRIC are written on tiles which are placed in a bag and then drawn out and arranged in order randomly. This procedure is repeated until an arrangement is obtained in which the two 'E's come together.

Find the probability that the procedure must be repeated exactly four times. [4]

4. Two spinners are in the form of an equilateral triangle, whose three regions are labelled 1, 2 and 3, and a square, whose four regions are labelled 1, 2, 3 and 4. Both spinners are biased and the probability distributions for the scores X and Y obtained when they are spun are respectively:

x	1	2	3		Y	1	2	3	4
$P(X=x)$	0·2	0·4	p		$P(Y=y)$	0·2	0·5	q	q

(i) Find the values of p and q . [2]

(ii) Find the probability that, when the two spinners are spun together, the sum of the two scores is (a) 5, (b) less than 4. [4]

(iii) State an assumption that you have made in answering part (ii) and explain why it is likely to be justifiable. [2]

(iv) Calculate $E(X)$ and $E(Y^2)$. [3]

5. In a survey for a computer magazine, the times t seconds taken by eight laser printers to print a page of text were compared with the prices $\pounds p$ of the printers. The data were coded using the equations $x = t - 10$ and $y = p - 150$, and it was found that

$$\Sigma x = 42.4, \quad \Sigma x^2 = 314.5, \quad \Sigma y = 560, \quad \Sigma y^2 = 60\,600, \quad \Sigma xy = 1592.$$

- (i) Find the mean time and the mean price for the eight printers. [4]
 (ii) Find the variance of the times. [3]
 (iii) Find the equation of the regression line of p on t . [5]
 (iv) Estimate the price of a printer which takes 11.3 seconds to print the page. [1]
6. 1000 houses were sold in a small town in a one-year period. The selling prices were as given in the following table:

Selling Price	Number of Houses	Selling Price	Number of Houses
Up to $\pounds 50\,000$	60	Upto $\pounds 250\,000$	650
Up to $\pounds 75\,000$	227	Upto $\pounds 400\,000$	800
Up to $\pounds 100\,000$	305	Upto $\pounds 500\,000$	900
Up to $\pounds 125\,000$	414	Upto $\pounds 800\,000$	1000

- (i) Draw a cumulative frequency graph to illustrate this data. [3]
 (ii) Use your graph to find estimates of the median and the quartiles. [4]
 (iii) Estimate the 37th percentile. [2]
- Given that the lowest price was $\pounds 42\,000$ and the range of the prices was $\pounds 690\,000$,
- (iv) draw a box-and-whisker plot to represent the data. [3]
- In another town the median price was $\pounds 149\,000$, and the interquartile range was $\pounds 90\,000$.
- (v) Briefly compare the prices in the two towns using this information. [2]

STATISTICS 1 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1. $\Sigma x = 12 \times 13 = 156$ B1
 $(\Sigma x^2)/12 - 13^2 = 10.2$ $\Sigma x^2 = 2150.4$ B1
 For whole set, $\Sigma x = 320$, $\Sigma x^2 = 4522.4$ Mean = 13.3 M1 A1
 Variance = $4522.4 \div 24 - 13.3^2 = 10.7$ M1 A1 6
2. (i) $P(A) \times P(B) = 3/8$ But $P(A \text{ and } B) = 7/20$, so not independent M1 A1
 (ii) $P(A) + P(C) = 7/10 = P(A \text{ AND } C)$, so mutually exclusive M1 A1
 (iii) $P(A \text{ AND } C) = 0$ as cannot occur at the same time B1 7
3. (i) $9! \div 2! = 181\,440$ ways M1 A1
 (ii) Group vowels so 5 letters + a group of 4 vowels (EEOI) = 6 things M1 A1
 No. of arrangements = $6 \times \frac{4!}{2!} = 8640$ as vowels arranged in $\frac{4!}{2!}$ ways
 $P(2 \text{ 'E's together}) = \frac{8!}{\binom{9!}{2}} = \frac{2}{9}$ as group E's to make 8 things.
 Total arrangements $\frac{9!}{2}$ as 2 E's
 $P(2 \text{ 'E's NOT together}) = 7/9$
 $P(2 \text{ E's occur on 4}^{\text{th}} \text{ attempt}) = 7/9 \times 7/9 \times 7/9 \times 2/9 = 0.105$ M1 A1 M1 A1 9
4. (i) $p = 0.4$ $2q = 0.3$ $q = 0.15$ B1 B1
 (ii) Using sample space or otherwise,
 (a) $P(\text{sum} = 5) = 0.03 + 0.06 + 0.2 = 0.29$ M1 A1
 (b) $P(\text{sum} < 4) = 0.04 + 0.1 + 0.08 = 0.22$ M1 A1
 (iii) Assumed independence. One is not likely to affect the other B1 B1
 (iv) $E(X) = 0.2 + 0.8 + 1.2 = 2.2$ B1
 $E(Y^2) = 0.2 + 2 + 1.35 + 2.4 = 5.95$ M1 A1 11
5. (i) $\Sigma t = \Sigma x + 80 = 122.4$ Mean time = $122.4 \div 8 = 15.3$ s M1 A1
 $\Sigma p = \Sigma y + 1200 = 1760$ Mean price = $\pounds 1760 \div 8 = \pounds 220$ M1 A1
 (ii) $\text{Var}(T) = \text{Var}(X + 10) = \text{Var}(X) = 314.5 \div 8 - 5.3^2 = 11.2$ M1 A1 A1
 (iii) y on x : $y - 70 = \frac{8(1592) - 42.4 \times 560}{8(314.5) - 42.4^2} (x - 5.3)$ M1 A1
 $y = -15.3x + 151.2$ $p - 150 = -15.3(t - 10) + 151.2$ M1 A1
 $p = -15.3t + 454$ (iv) $\pounds 281$ A1; A113
6. (i) Using given cumulative frequencies, graph drawn B3
 (ii) M1 A1 A1 A1
 (iii) M1 A1; B3
 (v) B1 B1 14