1. The points A, B and C have coordinates A(3, 2, –1), B(–1, 1, 2) and C(10, 5, –5), relative to the origin O.
Show that $\overrightarrow{OC}$ can be written in the form $\lambda\overrightarrow{OA} + \mu\overrightarrow{OB}$, where λ and μ are to be determined.

What can you deduce about the points O, A, B and C from the fact that $\overrightarrow{OC}$ can be expressed as a combination of $\overrightarrow{OA}$ and $\overrightarrow{OB}$?

[6]

2. In this question take $g = 10$.

The directions of the unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ are east, north and vertically upwards.

Forces **p, q** and **r** are given by $\mathbf{p} = \begin{pmatrix}-1\\-1\\5\end{pmatrix}$ N, $\mathbf{q} = \begin{pmatrix}-1\\-4\\2\end{pmatrix}$ N and $\mathbf{r} = \begin{pmatrix}2\\5\\0\end{pmatrix}$ N.

i. Find which of **p, q** and **r** has the greatest magnitude.

[2]

ii. A particle has mass 0.4 kg. The forces acting on it are **p, q, r** and its weight.

Find the magnitude of the particle's acceleration and describe the direction of this acceleration.

[4]

3. Show that points A (1, 4, 9), B (0, 11, 17) and C (3, –10, –7) are collinear. [4]

4. Points A, B and C have position vectors – **i** + 3**j** + 2**k** , 4**i** – **j** and 5**i** + **j** + 5**k** respectively.
Find the position vector of the point D such that ABCD is a parallelogram. [4]

5. ABCD is a parallelogram. The points A, B and C have position vectors

$$\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$$

respectively.

(a) Find the position vector of the point D. [3]

(b) Find the exact distance AC. [3]

6. Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where *a* and *b* can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with AC = AB.

(a) Show that $a - b + 1 = 0$. [4]

(b) Determine the position vector of A such that triangle ABC has minimum area. [6]

7. You are given that $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ and $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$.

Find a unit vector which is parallel to $\overrightarrow{\text{AB}}$. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1			$\begin{pmatrix}10\\5\\-5\end{pmatrix}=\lambda\begin{pmatrix}3\\2\\-1\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\2\end{pmatrix}$	M1	required form, can be soi from two or more correct equations
			$\Rightarrow 3\lambda - \mu = 10$	M1	forming at least two equations and attempting to solve oe
			$2\lambda + \mu = 5 \Rightarrow 5\lambda = 15$, $\lambda = 3$	A1	www
			$\Rightarrow 9 - \mu = 10$, $\mu = -1$	A1	www
			$-5 = -\lambda + 2\mu$, $-5 = -3 + 2 \times -1$ true	A1	verifying third equation, **do not** give BOD accept a statement such as $\begin{pmatrix}10\\5\\-5\end{pmatrix}=3\begin{pmatrix}3\\2\\-1\end{pmatrix}+-1\begin{pmatrix}-1\\1\\2\end{pmatrix}$ as verification Must **clearly** show that the solutions satisfy all the equations.
			coplanar	B1	oe **independent** of all above marks
					<u>Examiner's Comments</u> Most candidates scored the first four marks by forming the equations and solving them. Marks were usually lost both when candidates failed to show their solutions worked

					in all three equations or failed to realise that O, A, B and C must all lie on the same plane for the final mark.
			Total	6	
2		i	**p** $\sqrt{(-1)^2+(-1)^2+5^2}=\sqrt{27}$ **q** $\sqrt{(-1)^2+(-4)^2+2^2}=\sqrt{21}$ **r** $\sqrt{2^2+5^2+0^2}=\sqrt{29}$	M1	Use of Pythagoras Note Magnitudes are 5.196, 4.583 and 5.385 respectively
		i	Greatest magnitude: **r**	A1	Examiner's Comments Candidates were asked to find which of three forces, given as 3-dimensional column vectors, had the greatest magnitude. Almost all candidates got this right.
		ii	$\text{Weight} = \begin{pmatrix}0\\0\\-4\end{pmatrix}$	B1	Condone g = 9.8 giving weight is $\begin{pmatrix}0\\0\\-3.92\end{pmatrix}\text{N}$. Accept 4↓.
		ii	$\mathbf{p}+\mathbf{q}+\mathbf{r}+\mathbf{weight}=\begin{pmatrix}0\\0\\3\end{pmatrix}$		g = 9.8 gives $\begin{pmatrix}0\\0\\3.08\end{pmatrix}$
		ii	$0.4\mathbf{a}=\begin{pmatrix}0\\0\\3\end{pmatrix}$	B1	Relevant attempt at Newton's 2nd Law. The total force must be expressed as a vector in some form. For this mark allow the weight to be missing, in the wrong component or to have the wrong sign. Condone *mg* in place of *m* for this mark only.
		ii	Magnitude of acceleration is 7.5 ms^{-2}	B1	CAO apart from using g = 9.8 ⇒ a = 7.7

		ii	Direction is vertically upwards	B1	**Examiner's Comments** This question was about the application of Newton's second law to an object subject to the same three forces and its weight. Candidates needed to write the weight of the object in vector form. Many candidates got this completely right but others made mistakes with the weight, some applying it in the wrong direction or all three directions.
			Total	6	
3			$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 2 \\ -14 \\ -16 \end{pmatrix}$ AB is parallel to AC Common point A so collinear	M1(AO3.1a) A1(AO1.1) B1(AO1.1) E1(AO2.1) [4]	Attempt to find vector between any two of the points Correct pair of vectors with common point $\overrightarrow{BC} = \begin{pmatrix} 3 \\ -21 \\ -24 \end{pmatrix}$
			Total	4	
4			$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= 5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ $\overrightarrow{DC} = 5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ oe	M1(AO 1.1) A1(AO 1.1) M1(AO 3.1a)	Attempt to subtract two position vectors to get the vector for a side Equating opposite side Allow alternative vector notation

			$\overrightarrow{OD} = 5\mathbf{j} + 7\mathbf{k}$	A1(AO 2.2a) [4]	to vector found **or** finding $\overrightarrow{AC}$ and using $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$ oe — Allow $\overrightarrow{CD}$ here, but no ft from $\overrightarrow{CD}$ for final answer
			Total	4	
5		a	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$ $\overrightarrow{AD} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ $\overrightarrow{OD} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -5 \end{pmatrix}$	M1(AO3.1a) M1(AO3.1a) A1(AO3.1b) [3]	Use of $\|\|^{gm}$ and vector subtraction soi — $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$ $\overrightarrow{AD} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$
		b	$\overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}$ $AC = \sqrt{9+1+49}$ $= \sqrt{59}$	B1(AO1.1a) M1(AO1.1a) A1(AO1.1b) [3]	Or $\overrightarrow{CA} = \begin{pmatrix} 3 \\ -1 \\ -7 \end{pmatrix}$
			Total	6	

6		a	$\overrightarrow{AC}=\begin{pmatrix}2-a\\4-b\\2\end{pmatrix}$, $\overrightarrow{AB}=\begin{pmatrix}4-a\\2-b\\0\end{pmatrix}$ $(4-a)^2+(2-b)^2=(2-a)^2+(4-b)^2+4$ o.e. $16-8a+a^2+4-4b+b^2=4-4a+a^2+16-8b+b^2+4$ $4a-4b+4=0 \Rightarrow a-b+1=0$	M1 (AO 1.1) M1 (AO 1.1a) M1 (AO 1.1) A1 (AO 2.1) [4]	Forming vectors for sides AB and AC | Implied by next M1 Use of AB = AC expanding **AG** Convincing completion Examiner's Comments This part was generally answered well.
		b	D has position vector $\begin{pmatrix}3\\3\\1\end{pmatrix}$ where D is midpoint of BC	B1 (AO 3.1a) M1 (AO 1.1)	Midpoint OR if clearly minimising AC or AB – **M1** for relevant vector using a and b (May be implied by second M1) Finding relevant vector in terms of a or b only

			$\overrightarrow{AD} = \begin{pmatrix} 3-a \\ 2-a \\ 1 \end{pmatrix}$ $\text{Area} = \frac{1}{2} AD.BC = \frac{2\sqrt{3}\sqrt{(3-a)^2 + (2-a)^2 + 1}}{2}$ $\sqrt{3}\sqrt{2\left((a-2.5)^2 + 0.75\right)}$ *a* = 2.5 for min Position vector: $\begin{pmatrix} 2.5 \\ 3.5 \\ 0 \end{pmatrix}$	M1(AO 1.1) M1(AO 3.1a) A1(AO 2.2a) A1(AO 3.2a) [6]	Expression for AD or AD^2 (correct method but may have errors) Completion of square — May use area proportional to AD, AC or AB without calculation of expression for area Or differentiation of AD, AD^2, AC, AB, AC^2 or AB^2 <u>Examiner's Comments</u> This non-standard problem led to many different approaches but finding the midpoint of BC seemed the most logical start. The need to find a relevant vector such as A to the midpoint of *BC* in terms of one variable was appreciated by most candidates. The challenging part of this question was to find a for the minimum area and then use it to find the correct vector.
			Total	10	

7			b – *a* = 6i – 6j – 3**k**	M1 (AO1.1)	allow one slip	or in column vector form
			$\left\|AB\right\| = \sqrt{6^2 + (-6)^2 + (-3)^2}$	M1 (AO1.1)	FT their **b** – **a**	allow use of 2**i** – 2**j** – **k** oe
			⅔i – ⅔j – ⅓K	A1 (AO1.1) [3]	accept eg $^6/_9$**i** – $^6/_9$**j** – $^3/_9$**k**	or in column vector form
			Total	3		