Vectors Questions

7 The quadrilateral $ABCD$ has vertices $A\,(2, 1, 3)$, $B\,(6, 5, 3)$, $C\,(6, 1, -1)$ and $D\,(2, -3, -1)$.

The line l_1 has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that the line AB is parallel to l_1. *(1 mark)*

(iii) Verify that D lies on l_1. *(2 marks)*

(b) The line l_2 passes through $D\,(2, -3, -1)$ and $M\,(4, 1, 1)$.

(i) Find the vector equation of l_2. *(2 marks)*

(ii) Find the angle between l_2 and AC. *(3 marks)*

---

6 The points A and B have coordinates $(2,\ 4,\ 1)$ and $(3,\ 2,\ -1)$ respectively. The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$, where O is the origin.

(a) Find the vectors:

(i) $\overrightarrow{OC}$; *(1 mark)*

(ii) $\overrightarrow{AB}$. *(2 marks)*

(b) (i) Show that the distance between the points A and C is 5. *(2 marks)*

(ii) Find the size of angle BAC, giving your answer to the nearest degree. *(4 marks)*

(c) The point $P(\alpha,\ \beta,\ \gamma)$ is such that BP is perpendicular to AC.

Show that $4\alpha - 3\gamma = 15$. *(3 marks)*

---

6 The points A, B and C have coordinates $(3, -2, 4)$, $(5, 4, 0)$ and $(11, 6, -4)$ respectively.

(a) (i) Find the vector $\overrightarrow{BA}$. *(2 marks)*

(ii) Show that the size of angle ABC is $\cos^{-1}\left(-\frac{5}{7}\right)$. *(5 marks)*

(b) The line l has equation $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$.

(i) Verify that C lies on l. *(2 marks)*

(ii) Show that AB is parallel to l. *(1 mark)*

(c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of D. *(3 marks)*

---

7 The lines l_1 and l_2 have equations $\mathbf{r} = \begin{bmatrix} 8 \\ 6 \\ -9 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ respectively.

(a) Show that l_1 and l_2 are perpendicular. *(2 marks)*

(b) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection, P. *(5 marks)*

(c) The point $A(-4, 0, 11)$ lies on l_2. The point B on l_1 is such that $AP = BP$.

Find the length of AB. *(4 marks)*

Vectors Answers

7(a)(i)	$\overrightarrow{AB} = \begin{bmatrix}6\\5\\3\end{bmatrix} - \begin{bmatrix}2\\1\\3\end{bmatrix} = \begin{bmatrix}4\\4\\0\end{bmatrix}$	M1 A1	 2	Penalise use of co-ordinates at first occurrence only
(ii)	$\begin{bmatrix}4\\4\\0\end{bmatrix} = 4\begin{bmatrix}1\\1\\0\end{bmatrix} \Rightarrow$ parallel	E1	1	Needs comment "same direction" Or "same gradient" (**Or** by scalar product)
(iii)	$\begin{bmatrix}2\\-3\\-1\end{bmatrix} = \begin{bmatrix}6\\1\\-1\end{bmatrix} + \lambda\begin{bmatrix}1\\1\\0\end{bmatrix}$ is satisfied by $\lambda = -4$	M1 A1	 2	$\lambda = -4$ satisfies 2 equations
(b)(i)	l_2 has equation $\mathrm{r} = \begin{bmatrix}4\\1\\1\end{bmatrix} + \lambda\left[\begin{bmatrix}4\\1\\1\end{bmatrix} - \begin{bmatrix}2\\-3\\-1\end{bmatrix}\right] = \begin{bmatrix}4\\1\\1\end{bmatrix} + \lambda\begin{bmatrix}2\\4\\2\end{bmatrix}$	M1A1	2	**Or** $r = \begin{bmatrix}2\\-3\\-1\end{bmatrix} + t\begin{bmatrix}2\\4\\2\end{bmatrix}$ M1 calculate and use direction vector A1 all correct
(ii)	$\begin{bmatrix}1\\2\\1\end{bmatrix} \bullet \begin{bmatrix}4\\0\\-4\end{bmatrix} = 4 - 4 = 0$ $\Rightarrow 90°$ (or perpendicular)	M1A1 A1F	 3	Clear attempt to use directions of AC and l_2 in scalar product Accept a correct ft value of $\cos\theta$
	Total		**10**	

6(a)(i)	$\overrightarrow{OC} = 2\begin{bmatrix}3\\2\\-1\end{bmatrix} = \begin{bmatrix}6\\4\\-2\end{bmatrix}$	B1	1	(Penalise coordinates once only)
(ii)	$\overrightarrow{AB} = \begin{bmatrix}3\\2\\-1\end{bmatrix} - \begin{bmatrix}2\\4\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\-2\end{bmatrix}$	M1 A1	 2	$\overrightarrow{OA} - \overrightarrow{OB}$ or $\overrightarrow{OB} - \overrightarrow{OA}$ or 2/3 correct cpts. A0 for line AB
(b)(i)	$AC^2 = (6-2)^2 + (4-4)^2 + (-1-2)^2 = 25$ $AC = 5$	M1 A1	 2	$\lvert$Components of AC$\rvert$ AG
(ii)	$\overrightarrow{AB} \bullet \overrightarrow{AC} = \begin{bmatrix}1\\-2\\-2\end{bmatrix} \bullet \begin{bmatrix}4\\0\\-3\end{bmatrix} = 4 + 6 = 10$	M1 A1F		Clear attempt to use $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ft $\overrightarrow{AB}$ from a(ii) and/or $\overrightarrow{AC}$ from b(i)
	$3 \times 5 \times \cos\theta = 10$	M1		Use of $\lvert a \rvert \lvert b \rvert \cos\theta = \mathbf{a.b}$ with one correct $\lvert \; \rvert$ and **a.b** evaluated
	$\theta = 48.189 \approx 48°$	A1	4	CAO (AWRT)
	Alternative: use of cos rule Find 3rd side + use cos rule	(M2) (A1F) (A1)		 ft on previously found vectors CAO (AWRT)
(c)	$\overrightarrow{BP} = \begin{bmatrix}\alpha - 3\\\beta - 2\\\gamma - -1\end{bmatrix}$	B1		
	$\begin{bmatrix}4\\0\\-3\end{bmatrix} \bullet \overrightarrow{BP} = 0$	M1		Their $\overrightarrow{BP}$
	$4\alpha - 3\gamma - 15 = 0$	A1	3	AG convincingly obtained
	Total		**12**	

6(a)(i)	$\overrightarrow{BA}=\begin{bmatrix}3\\-2\\4\end{bmatrix}-\begin{bmatrix}5\\4\\0\end{bmatrix}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}$	M1A1	2	Attempt $\pm\overrightarrow{BA}$ $(OA-OB$ or $OB-OA)$
(ii)	$\overrightarrow{BC}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$	B1		Allow $\overrightarrow{CB}$; or $\begin{bmatrix}-6\\-2\\4\end{bmatrix}=\overrightarrow{BC}$ or $\overrightarrow{CB}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$ May not see explicitly
	$\lvert\overrightarrow{BA}\rvert\left(=\sqrt{(-2)^2+(-6)^2+(4)^2}\right)=\sqrt{56}$	B1F		Calculate modulus of $\overrightarrow{BA}$ or $\overrightarrow{BC}$; for finding modulus of one of vectors they have used
	$\overrightarrow{BA}\bullet\overrightarrow{BC}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}\bullet\begin{bmatrix}6\\2\\-4\end{bmatrix}=-12-12-16$	M1		Attempt at $\overrightarrow{BA}\bullet\overrightarrow{BC}$ with numerical answer; or $\overrightarrow{AB}\bullet\overrightarrow{CB}$
		A1		for –40, or correct if done with multiples of vectors
	$\cos ABC=\dfrac{-40}{\sqrt{56}\sqrt{56}}=-\dfrac{5}{7}$	A1	5	AG (convincingly obtained) Cosine rule: M1 attempt to find 3 sides A1 lengths of sides M1 cosine rule A1F correct A1 rearrange to get $\cos ABC=\dfrac{-5}{7}$ (ft on length of sides)
6 (cont) (b)(i)	$\begin{bmatrix}8\\-3\\2\end{bmatrix}+\lambda\begin{bmatrix}1\\3\\-2\end{bmatrix}=\begin{bmatrix}11\\6\\-4\end{bmatrix}$ $(\lambda=3)$	M1A1	2	$\lambda=3$ verified in three equations M1 for $\begin{cases}11=8+\lambda\\6=-3+3\lambda\\-4=2-2\lambda\end{cases}$ A1 for $\lambda=3$ shown for all three equations $\lambda\begin{bmatrix}1\\3\\-2\end{bmatrix}=\begin{bmatrix}11\\6\\-4\end{bmatrix}-\begin{bmatrix}8\\-3\\2\end{bmatrix}\therefore\lambda=3$ M1A1 SC: $\lambda=3$ written and nothing else: SC1
(ii)	$\begin{bmatrix}2\\6\\-4\end{bmatrix}=2\begin{bmatrix}1\\3\\-2\end{bmatrix}$ $\therefore$ same direction or same gradient or parallel	E1	1	

(c)	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$	B1		PI; $\overrightarrow{OD}$ = correct vector expression which may involve $\overrightarrow{AD}$
	$= \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -6 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$ D is $(9,0,0)$	M1A1	3	M1 for substituting into vector expression for $\overrightarrow{OD}$ NMS 3/3
	Total		**13**	

7(a)	$\begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = 3-6+3=0$	M1		attempt at sp, 3 terms, added
	$=0 \Rightarrow$ perpendicular	A1	2	$=0 \Rightarrow$ perpendicular seen $(\text{or } \cos\theta = 0 \Rightarrow \theta = 90^\circ)$ Allow $\begin{matrix} 3 \\ -6 \\ 3 \\ \hline 0 \end{matrix}$ but not $\begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix} = 0$
(b)	$8+3\lambda = -4+\mu$ $6-3\lambda = 2\mu$ $-9-\lambda = 11-3\mu$	M1		set up any two equations
	$\lambda = -2,\ \mu = 6$	m1 A1		solve for λ and μ
	verify third equation	m1		substitute λ, μ in third equation
	intersect at $(2,12,-7)$	A1	5	CAO
	Alt (for last two marks) substitute λ into l_1 and μ into l_2	(m1)		
	intersect at $(2,12,-7)$, condone $\begin{pmatrix} 2 \\ 12 \\ -7 \end{pmatrix}$	(A1)		$(2,12,-7)$ found from both lines Note: working for (b) done in (a): award marks in (b)
7(c)	$\overrightarrow{AP} = \begin{pmatrix} 6 \\ 12 \\ -18 \end{pmatrix}$	M1		$\overrightarrow{AP} = \pm\left\{ \text{their } \overrightarrow{OP} - \begin{pmatrix} -4 \\ 0 \\ 11 \end{pmatrix} \right\}$
	$AP^2 = 504$	A1F		ft on P
	$AB^2 = 2AP^2$	M1		Calculate AB^2
	$AB = 12\sqrt{7}$	A1	4	OE accept 31.7 or better
	Total		**11**	