Trigonometry Questions

3 It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of R. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of θ at which this maximum value occurs. *(3 marks)*

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6 (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where a and b are constants. *(2 marks)*

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4 (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*

(b) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

for all values of x. *(3 marks)*

(c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0° < x < 360°$. *(4 marks)*

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3 (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of x. *(2 marks)*

(ii) Solve the equation $3\sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*

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7 (a) Use the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$. *(2 marks)*

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of x, $\tan 2x \neq 0$. *(4 marks)*

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3 (a) Express $4\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving your value for α to the nearest 0.1°. *(3 marks)*

(b) Hence solve the equation $4\cos x + 3\sin x = 2$ in the interval $0° < x < 360°$, giving all solutions to the nearest 0.1°. *(4 marks)*

(c) Write down the minimum value of $4\cos x + 3\sin x$ and find the value of x in the interval $0° < x < 360°$ at which this minimum value occurs. *(3 marks)*

Trigonometry Answers

3(a)	$R=\sqrt{13}$ Or 3.6	B1	1	
(b)	$\frac{\sin\alpha}{\cos\alpha}=\tan\alpha=\frac{2}{3}$ $\alpha\approx 33.7$	M1A1	2	Allow M1 for $\tan\alpha=\frac{-2}{3}$ or $\pm\frac{3}{2}$ AG convincingly obtained
(c)	maximum value $=\sqrt{13}$	B1F		
	$\cos(\theta+33.7)=1$ $(\theta=-33.7)$	M1		
	$\theta=326.3$	A1	3	AWRT 326
	Total		**6**	

6(a)	$\cos 2x=2\cos^2 x-1$	B1B1	2	

4(a)(i)	$\sin 2x=2\sin x\cos x$	B1	1	
(ii)	$\cos 2x=2\cos^2 x-1$	B1	1	
(b)	$\sin 2x-\tan x=2\sin x\cos x-\frac{\sin x}{\cos x}$	M1		Use of their $\cos 2x$ or $\sin 2x$
	$=\sin x\left(2\cos x-\frac{1}{\cos x}\right)$	M1		Use of $\tan x=\frac{\sin x}{\cos x}$ and the other double angle identity
	$=\sin x\left(\frac{2\cos^2 x-1}{\cos x}\right)=\tan x\cos 2x$	A1	3	AG convincingly obtained
(c)	$\tan x\cos 2x=0$ $x=180$	B1		Ignore $x=0$, $x=360°$ & any others outside range
	$\cos 2x=0$ or $\cos^2 x=\frac{1}{2}$ $\left(\text{or } \sin^2 x=\frac{1}{2}\right)$	M1		
	$x=45$	A1		
	$x=135,225,315$	A1	4	CAO max 3/4 for answers in radians
	Total		**9**	

3(a)	$\cos 2x = 1 - 2\sin^2 x$	B1	1	
(b)(i)	$3\sin x - \cos 2x = 3\sin x - (1 - 2\sin^2 x)$	M1		Candidate's $\cos 2x$ or $\sin^2 x$
	$= 3\sin x - 1 + 2\sin^2 x$	A1	2	AG
(ii)	$2\sin^2 x + 3\sin x - 2 = 0$	M1		Soluble quadratic form
	$(2\sin x - 1)(\sin x + 2) = 0$	M1		Attempt to solve (allow one error in formula, allow sign errors)
	$\sin x = \frac{1}{2}$ $x = 30$ $x = 150$	M1		$\sin^{-1}$ and two solutions ($0° < x < 360°$)
		A1	4	A0 if radians
	Allow misread for			
	$2\sin^2 x + 3\sin x - 1 = 0$	(M1)		Soluble quadratic form
	$\sin x = \frac{-3 \pm \sqrt{17}}{4}$	(M1)		Use of formula (allow one error)
	$x = 16.3°, 163.7°$	(A1)		Max 3/4
(c)	$\int \frac{1}{2}(1 - \cos 2x) = \frac{x}{2} - \frac{\sin 2x}{4} (+c)$	M1A1	2	M1 – solve integral, must have 2 terms for $\sin^2 x$ from (a)
			9	

7(a)	$\tan(x + x) = \frac{\tan x + \tan x}{1 - \tan x \tan x} \left(= \frac{2\tan x}{1 - \tan^2 x} \right)$	M1		$A = B = x$ used
		A1	2	
(b)	$2 - 2\tan x - \frac{2\tan x(1 - \tan^2 x)}{2\tan x}$	M1		Substitute from (a)
	$2 - 2\tan x - (1 - \tan x)(1 + \tan x)$	M1		Simplification $2 - 2\tan x - (1 - \tan^2 x)$
	$(1 - \tan x)(2 - (1 + \tan x))$	M1		$2 - 2\tan x - 1 + \tan^2 x$
	$(1 - \tan x)^2$	A1	4	AG (convincingly obtained)
				$= (\tan x - 1)^2 = (1 - \tan x)^2$ Any equivalent method
	Total		**6**	

3(a)	$R=5$	B1		
	$\tan\alpha=\frac{3}{4}$ (OE) $\alpha=36.9°$ (ISW 216.9)	M1A1	3	SC1 $\tan\alpha=\frac{4}{3}, \alpha=53.1°$ R,α PI in (b)
(b)	$\cos(x-\alpha)=\frac{2}{R}$	M1		
	$x-\alpha=66.4°$	A1		
	$x=103.3°$	A1F		
	$x=330.4°$	A1F	4	accept $330.5°$, -1 each extra ft on acute α
(c)	minimum value $=-5$	B1F		ft on R
	$\cos(x-36.9)=-1$	M1		SC $\cos(x+36.9)$ treat as miscopy
	$x=216.9°$	A1	3	216.9 or better accept graphics calculator solution to this accuracy SC Find max: max = 5 at $(x+36.9)$ stated 1/3 Max 8/10 for work in radians
	Total		**10**	