

Trigonometry Questions

- 1 (a) Find $\frac{dy}{dx}$ when $y = \tan 3x$. (2 marks)
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4 It is given that $2\operatorname{cosec}^2 x = 5 - 5 \cot x$.

- (a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5 \cot x$ can be written in the form

$$2 \cot^2 x + 5 \cot x - 3 = 0 \quad (2 \text{ marks})$$

- (b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. (2 marks)

- (c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5 \cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leq \pi$. (3 marks)
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- 3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leq x \leq 2\pi$ in radians to two decimal places. (3 marks)

- (b) Show that the equation $\tan^2 x = 3 \sec x + 9$ can be written as

$$\sec^2 x - 3 \sec x - 10 = 0 \quad (2 \text{ marks})$$

- (c) Solve the equation $\tan^2 x = 3 \sec x + 9$, giving all the values of x in the interval $0 \leq x \leq 2\pi$ in radians to two decimal places. (4 marks)
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- 7 (a) Given that $z = \frac{\sin x}{\cos x}$, use the quotient rule to show that $\frac{dz}{dx} = \sec^2 x$. (3 marks)

- (b) Sketch the curve with equation $y = \sec x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. (2 marks)
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- (b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2} \sin y$. (1 mark)

- (ii) Given that $x = \frac{1}{2} \sin y$, find $\frac{dx}{dy}$ in terms of y . (1 mark)

- (c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} \quad (4 \text{ marks})$$

- 2 Describe a sequence of two geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. (4 marks)
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- 5 (a) (i) Show that the equation

$$2 \cot^2 x + 5 \operatorname{cosec} x = 10$$

can be written in the form $2 \operatorname{cosec}^2 x + 5 \operatorname{cosec} x - 12 = 0$. (2 marks)

- (ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. (3 marks)

- (b) Hence, or otherwise, solve the equation

$$2 \cot^2(\theta - 0.1) + 5 \operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. (3 marks)

- 6 (a) Find $\frac{dy}{dx}$ when:

(ii) $y = x^2 \tan x$. (2 marks)

- 8 (a) Write down $\int \sec^2 x \, dx$. (1 mark)

- (b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. (4 marks)

- (c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. (3 marks)

- (d) Hence find $\int_{0.5}^1 (\tan x + \cot x)^2 \, dx$, giving your answer to two significant figures. (4 marks)

Trigonometry Answers

1(a)	$\frac{dy}{dx} = 3\sec^2 3x$ <p>Alternative Use of product/Quotient rule (M1) $\frac{3\cos^2 3x + 3\sin^2 3x}{\cos^2 3x}$ (A1)</p>	M1 A1	2	for sec 3x SC/3sec ² x B1 Good attempt Correct						
4(a)	$2\operatorname{cosec}^2 x = 5(1 - \cot x)$ $2 + 2\cot^2 x = 5 - 5\cot x$ $2\cot^2 x + 5\cot x - 3 = 0$	M1 A1	2	use of cosec ² x = 1 + cot ² x AG						
(b)	$(2\cot x - 1)(\cot x + 3) = 0$ $\cot x = \frac{1}{2}, -3$ $\tan x = 2, -\frac{1}{3}$	M1 A1	 2	or $2 + 5t - 3t^2 = 0$ Or in tan x $(2 - t)(1 + 3t) = 0$ AG						
(c)	$\left. \begin{array}{l} x = 1.1, -2.0 \\ x = -0.3, 2.8 \end{array} \right\} \text{AWRT}$	B1 B1 B1	 3	<table style="border: none;"> <tr> <td style="border: none;">Any 2 correct</td> <td style="border: none;">] In degrees: B0</td> </tr> <tr> <td style="border: none;">Any 3 correct</td> <td style="border: none;">] B1</td> </tr> <tr> <td style="border: none;">4 correct</td> <td style="border: none;">] B2</td> </tr> </table>	Any 2 correct] In degrees: B0	Any 3 correct] B1	4 correct] B2
Any 2 correct] In degrees: B0									
Any 3 correct] B1									
4 correct] B2									
3(a)	$\sec x = 5$ $\cos x = 0.2$ $x = 1.37, 4.91 \text{ AWRT}$	M1 A1A1	3							
(b)	$\tan^2 x = 3\sec x + 9$ $\sec^2 x - 1 = 3\sec x + 9$ $\sec^2 x - 3\sec x - 10 = 0$	M1 A1	2	for using sec ² x = 1 + tan ² x OE AG						
(c)	$(\sec x - 5)(\sec x + 2) = 0$ $\sec x = 5, -2$ $\cos x = 0.2, -0.5$ $x = 1.37, 4.91$ $2.09, 4.19$	M1 A1 B1F A1	 4	or use of formula (attempt) any 2 correct or fit their 2 answers in (a) all 4 correct, no extras						

7(a)	$z = \frac{\sin x}{\cos x}$ $\frac{dz}{dx} = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$ $= \frac{1}{\cos^2 x}$ $= \sec^2 x$	M1 A1		use of quotient rule $\left(\frac{\pm \cos^2 x \pm \sin^2 x}{\cos^2 x}\right)$
(b)		A1	3	AG (be convinced)
		M1		correct shape including asymptotic behaviour and symmetrical about $x = 0$ and $y > 0$
		A1	2	use of 1

(b)(i)	$y = \sin^{-1} 2x$ $\sin y = 2x$ and $\frac{1}{2} \sin y = x$	B1	1	AG (be convinced)
(ii)	$\frac{dx}{dy} = \frac{1}{2} \cos y$	B1	1	
(c)	$\frac{dy}{dx} = \frac{2}{\cos y}$ $\sin y = 2x$ and $\sin^2 + \cos^2 = 1$ $\cos y = \sqrt{1 - 4x^2}$ $\frac{dy}{dx} = \frac{2}{\sqrt{1 - 4x^2}}$	M1A1 M1 A1		M1 for $\frac{k}{\cos y}$ use of to get $\cos y$ or $\cos^2 y$ AG; condone omission of proof of sign

2	Stretch (I) SF $\frac{1}{3}$ (II) Parallel to x - axis (III) Translate $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	M1 A1 E1 B1		For I + (II or III) All correct Allow translation Correct vector or description
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5(a)(i)	$2(\operatorname{cosec}^2 x - 1) + 5 \operatorname{cosec} x = 10$	M1	2	AG
	$2 \operatorname{cosec}^2 x - 2 + 5 \operatorname{cosec} x - 10 = 0$	A1		
(ii)	$2 \operatorname{cosec}^2 x + 5 \operatorname{cosec} x - 12 = 0$	M1	3	AG
	$(2 \operatorname{cosec} x - 3)(\operatorname{cosec} x + 4) = 0$	A1		
	$\operatorname{cosec} x = \frac{3}{2}$ or -4	A1		
(b)	$\sin x = \frac{2}{3}$ or $-\frac{1}{4}$	A1	3	AG
	$(\theta - 0.1) = 0.73, 2.41, -0.25, -2.89$	B1		
	$\theta = 0.83, 2.51, -0.15, -2.79$ AWRT	B1		
	$\theta = 0.83, 2.51, -0.15, -2.79$ AWRT	B1		

(ii)	$y = x^2 \tan x$	M1	2	Product rule
	$\frac{dy}{dx} = x^2 \sec^2 x + 2x \tan x$	A1		

8(a)	$\tan x$ (+ c)	B1	1	
(b)	$f(x) = \frac{\cos x}{\sin x}$	M1 A1 A1 A1	4	AG CSO Special cases $f(x) = \frac{\cot x}{1}$ $f'(x) = \frac{1 \times -\operatorname{cosec}^2 x - \cot x \times 0}{1^2}$ M1 $= -\operatorname{cosec}^2 x$ A1 (max 2/4) Or $f(x) = \frac{1}{\tan x}$ $f'(x) = \frac{\tan x \times 0 - 1 \times \sec^2 x}{\tan^2 x}$ M1 A1 $= \frac{-\sec^2 x}{\tan^2 x}$ $= \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2$ A1 (max 3/4)
	$f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$			
	$= \frac{-1}{\sin^2 x}$			
	$= -\operatorname{cosec}^2 x$			

(c)	$\begin{aligned} \text{LHS} &= \tan^2 x + \cot^2 x + 2 \tan x \cot x \\ &= \tan^2 x + 1 + \cot^2 x + 1 \\ &= \sec^2 x + \text{cosec}^2 x \\ &= \text{RHS} \end{aligned}$	M1 M1 A1	3	expanding correct use of trig identities CSO
(d)	$\begin{aligned} \int (\tan x + \cot x)^2 dx &= \int \sec^2 x + \text{cosec}^2 x dx \\ &= [\tan x - \cot x]_{0.5}^1 \\ &= 0.9153 - -1.2842 \\ &= 2.2 \end{aligned}$	M1 M1 A1 A1	4	use of identity $\pm \tan x \pm \cot x$ OE AWRT