VECTORS — Answers

1 a) $-\mathbf{p}$ b) $2\mathbf{q}$ c) $\frac{1}{2}\mathbf{p}$ d) $\mathbf{p}$ e) $-\mathbf{q}$ f) $\mathbf{p}+\mathbf{q}$

g) $\frac{1}{2}\mathbf{p}+2\mathbf{q}$ h) $\mathbf{p}-\mathbf{q}$ i) $2\mathbf{q}-\mathbf{p}$ j) $-\mathbf{p}-2\mathbf{q}$ k) $\frac{1}{2}\mathbf{p}-\mathbf{q}$ l) $-\frac{1}{2}\mathbf{p}-2\mathbf{q}$

2 a) $\mathbf{u}+\mathbf{v}$ b) $\mathbf{w}-\mathbf{u}$ c) $\mathbf{u}+\mathbf{v}-\mathbf{w}$

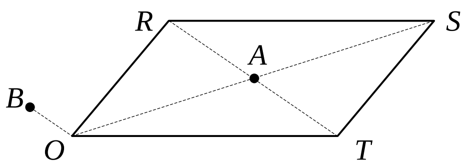
3 a) $\mathbf{q}$ b) $\mathbf{p}+\mathbf{r}$ c) $\mathbf{r}-\mathbf{q}$ d) $\mathbf{p}+\mathbf{q}+\mathbf{r}$ e) $-\mathbf{q}-\mathbf{r}$ f) $\mathbf{q}+\mathbf{r}-\mathbf{p}$

4 **a** **i** $= (\mathbf{a}+2\mathbf{b}) + (\mathbf{a}-2\mathbf{b})$
$= 2\mathbf{a}$

ii $= (\mathbf{a}+2\mathbf{b}) - (\mathbf{a}-2\mathbf{b})$
$= 4\mathbf{b}$

b $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{OS}$, $\overrightarrow{OB} = \frac{1}{4}\overrightarrow{TR}$

$\therefore$



5 **a** **i** $= \frac{1}{2}\mathbf{a}$

ii $= \mathbf{b}-\mathbf{a}$

iii $= \frac{1}{2}(\mathbf{b}-\mathbf{a})$

iv $= \mathbf{a} + \frac{1}{2}(\mathbf{b}-\mathbf{a}) = \frac{1}{2}(\mathbf{a}+\mathbf{b})$

v $= \frac{1}{2}(\mathbf{a}+\mathbf{b}) - \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{b}$

b they are parallel (and the magnitude of $\overrightarrow{CD}$ is half that of $\overrightarrow{OB}$)

6 **a** parallel, $3\mathbf{p} = \frac{3}{2}(2\mathbf{p})$

b not parallel

c parallel, $(\mathbf{p} - \frac{1}{3}\mathbf{q}) = \frac{1}{3}(3\mathbf{p}-\mathbf{q})$

d parallel, $(4\mathbf{q}-2\mathbf{p}) = -2(\mathbf{p}-2\mathbf{q})$

e parallel, $(6\mathbf{p}+8\mathbf{q}) = 8(\frac{3}{4}\mathbf{p}+\mathbf{q})$

f not parallel

7 **a** $= (2\mathbf{m}+3\mathbf{n}) - (4\mathbf{m}+2\mathbf{n})$
$= \mathbf{n}-2\mathbf{m}$

b $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC} = \mathbf{m} + \frac{3}{2}\mathbf{n}$

$\overrightarrow{AM} = (\mathbf{m}+\frac{3}{2}\mathbf{n}) - 4\mathbf{m} = \frac{3}{2}\mathbf{n} - 3\mathbf{m}$

$\therefore\ \overrightarrow{AM} = \frac{3}{2}\overrightarrow{BC}$

$\therefore$ *AM* is parallel to *BC*

8 **a** $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} = 3\mathbf{u} - 2\mathbf{v}$

$\overrightarrow{AB} = (3\mathbf{u} - \mathbf{v}) - (6\mathbf{u} - 4\mathbf{v}) = 3\mathbf{v} - 3\mathbf{u}$

$\overrightarrow{ON} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB} = (6\mathbf{u} - 4\mathbf{v}) + \frac{1}{3}(3\mathbf{v} - 3\mathbf{u}) = 5\mathbf{u} - 3\mathbf{v}$

b $\overrightarrow{CM} = (3\mathbf{u} - 2\mathbf{v}) - (\mathbf{v} - 3\mathbf{u}) = 6\mathbf{u} - 3\mathbf{v}$

$\overrightarrow{CN} = (5\mathbf{u} - 3\mathbf{v}) - (\mathbf{v} - 3\mathbf{u}) = 8\mathbf{u} - 4\mathbf{v}$

$\therefore\ \overrightarrow{CN} = \frac{4}{3}\overrightarrow{CM}$

$\therefore\ \overrightarrow{CN}$ and $\overrightarrow{CM}$ are parallel
common point C $\therefore$ C, M and N are collinear

9 **a** $a = 5,\ b = 3$

b $2 + b = 0$ and $a - 4 = 0$
$\therefore\ a = 4,\ b = -2$

c $-1 = b$ and $4a = -2$
$\therefore\ a = -\frac{1}{2},\ b = -1$

d $2a + 6 = 0$ and $b - a = 0$
$\therefore\ a = -3,\ b = -3$

10 **a** $\overrightarrow{OC} = \frac{1}{2}\mathbf{a}$

$\overrightarrow{CB} = \mathbf{b} - \frac{1}{2}\mathbf{a}$

$\overrightarrow{OD} = \frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{b} - \frac{1}{2}\mathbf{a}) = \frac{1}{4}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{OE} = \overrightarrow{OA} + k\overrightarrow{AB}$

$\therefore\ \overrightarrow{OE} = \mathbf{a} + k(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OE} = l\overrightarrow{OD}$

$\therefore\ \mathbf{a} + k(\mathbf{b} - \mathbf{a}) = l(\frac{1}{4}\mathbf{a} + \frac{1}{2}\mathbf{b})$

$\therefore$ $1 - k = \frac{1}{4}l$

and $k = \frac{1}{2}l$

adding $1 = \frac{3}{4}l$

$l = \frac{4}{3}$

$\therefore\ \overrightarrow{OE} = \frac{4}{3}(\frac{1}{4}\mathbf{a} + \frac{1}{2}\mathbf{b}) = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

d $k = \frac{1}{2}l = \frac{2}{3}$

$\therefore\ \overrightarrow{AE} = \frac{2}{3}\overrightarrow{AB}$

$\therefore\ AE : EB = 2 : 1$