1.

The points A, B and C have position vectors $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ respectively. M is the midpoint of BC.

(a) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. [3]

(b) Find the magnitude of $\overrightarrow{AM}$. [3]

2.

The point A has position vector i – 2j. The point B is such that $\left|\overrightarrow{OB}\right| = \left|\overrightarrow{OA}\right|$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OA}$.

(a) (i) Find $\left|\overrightarrow{OB}\right|$. [2]

(ii) Find the two possible directions of $\overrightarrow{OB}$, giving your answers correct to the nearest [2] degree.

The point C is such that $\left|\overrightarrow{AC}\right| = 2$.

(b) Find the maximum and minimum values of $\left|\overrightarrow{OC}\right|$. [4]

3. Vectors a and b are defined as follows: a = 2i + 6j and b = 2i – 4j.
(a) Given that pa + qb = 6i – 7j, find the values of the constants p and q. [3]

(b) It is now given instead that |a + kb| = 5. Use the diagram below to find the two possible values of the constant k. [4]
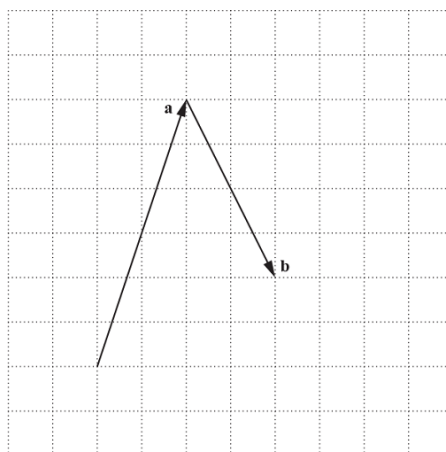
4.

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. $P$ is the midpoint of $AC$.
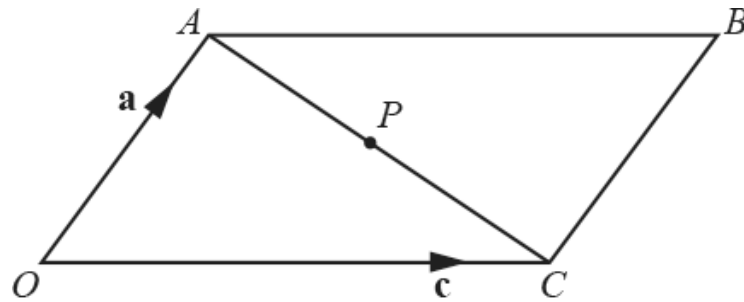


(a) Find the following in terms of **a** and **c**, simplifying your answers.

(i) $\overrightarrow{AC}$ [1]

(ii) $\overrightarrow{OP}$ [2]

(b) Hence prove that the diagonals of a parallelogram bisect one another. [4]

5. Vector **v** = $a\mathbf{i}$ + 0.6**j**, where $a$ is a constant.

(a) Given that the direction of **v** is 45°, state the value of $a$. [1]

(b) Given instead that **v** is parallel to 8**i** + 3**j**, find the value of $a$. [2]

(c) Given instead that **v** is a unit vector, find the possible values of $a$. [3]

END OF QUESTION paper

# Mark scheme

| Question | | | Answer/Indicative content | Marks | Guidance |
|---|---|---|---|---|---|
| 1 | a | | $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \mathbf{d} - \mathbf{a} = \overrightarrow{AD}$ $\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ | B1(AO1.1) M1(AO3.1a) A1(AO1.1) [3] | soi |
| | b | | $\overrightarrow{OM} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ $\left| \overrightarrow{AM} \right| = \sqrt{6^2 + 3^2} = 3\sqrt{5}$ | B1(AO1.1) M1(AO1.1) A1(AO2.2a) [3] | soi Accept 6.71 |
| | | | Total | 6 | |
| 2 | a | i) | $\left| \overrightarrow{OB} \right| = \sqrt{1^2 + 2^2}$ Mag $= \sqrt{5}$ or 2.24 (3 sf) | M1(AO1.2) A1(AO1.1) [2] | |
| | a | | ii) Direction ($= \tan^{-1}(0.5)$) $= 27°$ & ($180° + 27°$ or $\tan^{-1}(-0.5)$) $= 207°$ | M1(AO1.1a) A1f(AO1.1) [2] | ft their 27° |
| | b | | For max & min $OC$, $C$ lies on $OA$ $OC = OA \pm 2$ Max $OC = \sqrt{5} + 2$ or 4.24 (3 sf) Min $OC = \sqrt{5} - 2$ or 0.236 (3 sf) | M1(AO2.1) M1(AO3.1a) A1(AO2.2a) A1(AO1.1) [4] | May be implied, eg by diagram Their $OA$ (from (a)) $\pm 2$ |
| | | | Total | 8 | |
| 3 | a | | $2p + 2q = 6$ $6p - 4q = -7$ eg $4p + 4q = 12$ | B1(AO3.1a) | |

| | | | | | |
|---|---|---|---|---|---|
| | | $10p = 5$ <br><br><br> $p = 0.5$, $q = 2.5$ | M1(AO 1.1) <br><br> A1(AO 1.1) <br><br> [3] | Both <br><br> Correct method to solve and achieve any correct equation in either $p$ or $q$ <br><br><br><br><br> Both | |

| | | | | | |
|---|---|---|---|---|---|
| | b | Vectors 3**i** + 4**j** and 5**i** shown on diagram, each starting at start point of vector **a** <br><br> $k = 0.5$ <br><br> or 1.5 | (AO1.2) <br> B1B1(AO1.1) <br><br> B1(AO2.2a) <br><br> B1(AO1.1) <br><br> [4] | or just end points of these vectors shown | |

| | | | | | |
|---|---|---|---|---|---|
| | | **Total** | **7** | | |

| | | | | | |
|---|---|---|---|---|---|
| 4 | ` | Allow without arrows or squiggles throughout | |  <br><br> **Examiner's Comments** <br><br> In all three parts of this question, many candidates did not use correct vector notation. | |

| | | | | | |
|---|---|---|---|---|---|
| | a | **(i)**    $c - a$       oe | B1 <br> (AO1.2) <br><br> [1] |  <br><br> **Examiner's Comments** <br><br> Almost all candidates answered this question correctly. | |
| | a | **(ii)**   $\mathbf{a} + \dfrac{1}{2}(\mathbf{c} - \mathbf{a})$   or   $\mathbf{c} + \dfrac{1}{2}(\mathbf{a} - \mathbf{c})$ <br><br> $= \dfrac{1}{2}(\mathbf{a} + \mathbf{c})$   or   $\dfrac{1}{2}\mathbf{a} + \dfrac{1}{2}\mathbf{c}$ | M1 <br> (AO3.1a) <br><br><br> A1 <br> (AO1.1b) <br><br> [2] | $\mathbf{a} + \dfrac{1}{2}$ their (i) <br> or   $\mathbf{c} - \dfrac{1}{2}$ their (i) <br><br> Correct ans without wking: M1A1 | |

Most answered this question correctly. A few made a sign error, for

example $c + \frac{1}{2}(c - a)$ .

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**b**

$$\overrightarrow{OB} = (a + c)$$

$$\Rightarrow \overrightarrow{OP} = \frac{1}{2}\overrightarrow{OB}$$

Must see previous line

$\Rightarrow$ P is midpt of OB

or OPB is a straight line and OP = PB

Hence diagonals of /m bisect one another

| | |
|---|---|
| M1 (AO3.1a) | $\overrightarrow{PB} = a + \frac{1}{2}(c - a)$ <br> or $a + \frac{1}{2}$ their(a)(i) <br> or $c + \frac{1}{2}(a - c)$ <br> ($= \frac{1}{2}(a+c)$ oe) <br> ft their (a)(i)    or $\overrightarrow{PB} = c - \frac{1}{2}$ their(a)(i) |
| A1* (AO1.1) <br><br> dep*A1 (AO2.1) <br> E1 (AO2.2a) <br><br> [4] | NB $\overrightarrow{PB} = \frac{1}{2}(a + c)$ without justification: M0A0A0E0 <br><br> $\Rightarrow \overrightarrow{PB} = \overrightarrow{OP}$ <br> dep M1A1A1    or similar with $\overrightarrow{BP}$ or $\overrightarrow{BO}$ |

**Examiner's Comments**

This question proved challenging for a significant majority of

candidates. Many assumed the result by starting with, for example,

$PB = \frac{1}{2}(c + a)$ , instead of deriving this result. Some

candidates considered the modulus of some vectors. Some

candidates seemed unaware of the meaning of the word "bisect", in

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| | | | | | |
|---|---|---|---|---|---|
| | | | | some cases confusing it with "perpendicular". Thus many wrote that a + c is perpendicular to a – c, and that this somehow proves that the diagonals bisect one another. Perhaps the majority of candidates did not know how to start answering this question at all.<br><br>An example of a candidate's solution that suggested they had no understanding of proof by vectors was as follows:<br><br>"BO = AC. As they are the same length it means they would both meet in the centre, hence meaning they bisect one another." | |
| | | Total | 7 | | |
| 5 | a | $a = 0.6$ | B1 (AO 1.2)<br><br>[1] | State correct value for $a$ | |
| | b | $3k = 0.6$, so $k = 0.2$<br><br>$a = 8 \times 0.2 = 1.6$ | M1 (AO 1.1a)<br><br>A1 (AO 1.1)<br><br>[2] | Attempt to find scale factor<br><br>Obtain $a = 1.6$ | OR $0.6k = 3$, so $k = 5$ |
| | c | $\sqrt{a^2 + 0.6^2} = 1$<br><br>$a^2 = 0.64$<br><br>$a = \pm 0.8$ | B1 (AO 1.2)<br><br>M1 (AO 1.1a)<br><br>A1 (AO 1.1)<br><br>[3] | Correct definition for unit vector seen or implied<br><br>Attempt to find at least one value for $a$<br><br>Both correct values for $a$ | Allow BOD for $a^2 + 0.6^2 = 1$, with no square root seen |
| | | Total | 6 | | |