1. The unit vectors **i** and **j** shown in Fig. 2 are in the horizontal andvertically upwards directions.

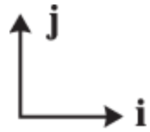

Fig. 2

Forces **p** and **q** are given, in newtons, by **p** = 12**i** – 5**j** and **q** = 16**i** + 1.5**j**.

i. Write down the force **p** + **q** and show that it is parallel to 8**i** – **j**.

[3]

ii. Show that the force 3**p** + 10**q** acts in the horizontal direction.

[2]

iii. A particle is in equilibrium under forces k**p**, 3**q** and its weight **w**.

Show that the value of k must be –4 and find the mass of the particle.

[3]

2. Two points, A and B, have position vectors **a** = **i** – 3**j** and **b** = 4**i** + 3**j** . The point C lies on the line $y = 1$. The lengths of the line segments AC and BC are equal. Determine the position vector of C. [4]

3. OACB is a parallelogram. O is the origin and point A has coordinates (5, 6). Point B has position vector
b = –2**i** – 7**j**.

(a) Find the coordinates of point C. [3]

M is the midpoint of AB.

(b) Prove that $\overrightarrow{OM} = \overrightarrow{MC}$. [3]

(c) Find the exact distance MC. [2]

END OF QUESTION paper

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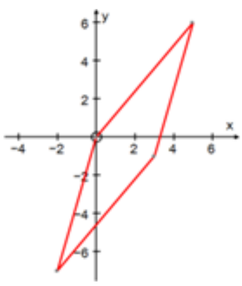
Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		i	**p** + **q** = 28**i** – 3.5**j**	B1	
		i	28**i** – 3.5**j** = *k*(8**i** – **j**)	M1	Or equivalent. *k* may be implied by going straight to 3.5
			k = 3.5		
		i	(So they are parallel)	A1	
			Alternative		
			p + **q** = 28**i** – 3.5**j**		
		i	$\mathbf{p}+\mathbf{q}: \quad \tan\theta = \frac{-3.5}{28} \quad \Rightarrow \quad \theta = -7.13°$	B1	
		i	$8\mathbf{i}-\mathbf{j}: \quad \tan\theta = \frac{-1}{8} \quad \Rightarrow \quad \theta = -7.13°$	M1	Comparing the ratio of the components in each of the two vectors is sufficient, using any consistent sign convention. The angle does not need to be worked out, nor does tan have to be seen.
		i	So they are parallel	A1	Both ratios the same and correct
		ii	3**p** + 10**q** = (36 + 160)**i** + (–15 + 15)**j**		
		ii	=196**i**	B1	
		ii	Zero **j** component so horizontal	B1	Or equivalent explanation. May be shown on a diagram
		iii	The horizontal component must be zero		Substituting *k* = –4 and showing **i** component is zero is acceptable Award for 24.5 seen Award for 2.5 seen. FT from their weight.
		iii	So 12*k* + 3 × 16 = 0 ⇒ *k* = –4	B1	Substituting *k* = –4 and showing **i** component is zero is acceptable
		iii	**w** = –24.5**j**	B1	Award for 24.5 seen
		iii	*mg* = 24.5 ⇒ *m* = 2.5 The mass is 2.5 kg	B1	Award for 2.5 seen. FT from their weight. **Examiner's Comments** This question was about vectors defined using i, j notation. It was well answered with many candidates obtaining full marks. Most of the errors that did occur were in part (iii) where candidates were required to interpret the vectors as forces on a particle in equilibrium, with its weight now introduced. Some did

					not interpret the equilibrium conditions sufficiently rigorously, failing to recognise that both the horizontal and vertical components of the resultant had to be zero. The question ended with a request for the mass of the particle and some candidates confused this with its weight.
			Total	8	
2			Midpoint of $AB = \left(\frac{5}{2}, 0\right)$ Gradient of perpendicular to $AB = -\frac{1}{2}$ Perpendicular bisector equation is $y = -\frac{1}{2}\left(x - \frac{5}{2}\right)$ oe Position vector is $\frac{1}{2}\mathbf{i} + \mathbf{j}$ **Alternative method** Suppose C has position vector **c** = p**i** + **j** $\overrightarrow{\mathbf{AC}}$ = (p –1)**i** + 4**j** oe or $\|AC\|^2 = (p-1)^2 + 4^2$ oe $\overrightarrow{\mathbf{BC}}$ = (p – 4)**i** – 2**j** oe or $\|BC\|^2 = (p-4)^2 + 2^2$ oe $(p-1)^2 + 4^2 = (p-4)^2 + 2^2$ Position vector is $\frac{1}{2}\mathbf{i} + \mathbf{j}$	M1(AO3.1a) M1(AO1.1) A1(AO1.1) A1(AO1.1) [4] M1(AO3.1a) M1(AO1.1) A1(AO1.1) A1(AO1.1)	soi
			Total	4	
3		a	 **a** = 5**i** + 6**j** $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{OA} = \mathbf{b} + \mathbf{a}$ $= (5\mathbf{i} + 6\mathbf{j}) + (-2\mathbf{i} - 7\mathbf{j}) = 3\mathbf{i} - \mathbf{j}$	M1(AO 1.1a) M1(AO 3.1a)	Using vector notation Using **a** for

			So C is (3, −1)	E1(AO 3.2a) [3]	vector $\overrightarrow{\mathrm{BC}}$ in an addition Must be coordinates; do not allow for position vector only
		b	$\overrightarrow{\mathrm{OM}} = \frac{1}{2}(\mathbf{a}+\mathbf{b}) = \frac{3}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ $\overrightarrow{\mathrm{MC}} = (3\mathbf{i}-\mathbf{j}) - \left(\frac{3}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$ $= \left(\frac{3}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right) = \overrightarrow{\mathrm{OM}}$	B1(AO 2.1) M1(AO 1.1a) A1(AO 2.1) [3]	
		c	$\lvert\overrightarrow{\mathrm{MC}}\rvert = \left\lvert \frac{3}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} \right\rvert = \frac{1}{2}\sqrt{3^2+(-1)^2}$ $= \frac{1}{2}\sqrt{10}$	M1(AO 1.1a) A1(AO 1.1b) [2]	Or exact equivalent, e.g. $\sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$ Accept alternative answer $\sqrt{\frac{5}{2}}$
			Total	8	