Questions

Q1.

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for question = 4 marks)

Q2.

Given that the point *A* has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point *B* has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

(Total for question = 4 marks)

Q3.

(i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$

The angle between vector **m** and vector **n** is 30°
Find the angle between vector **m** and vector **m – n**, giving your answer, in degrees, to one decimal place.

(4)

(Total for question = 5 marks)

Q4.

[*In this question the unit vectors* **i** and **j** *are due east and due north respectively*.]

A coastguard station *O* monitors the movements of a small boat.

At 10:00 the boat is at the point (4**i** − 2**j**) km relative to *O*.

At 12:45 the boat is at the point (−3**i** − 5**j**) km relative to *O*.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

(Total for question = 6 marks)

Q5.

[*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

(Total for question = 5 marks)

Q6.

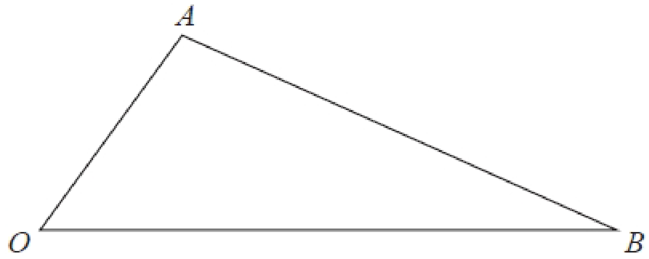


Figure 7

Figure 7 shows a sketch of triangle *OAB.*

The point *C* is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point *M* is the midpoint of *AB*.

The straight line through *C* and *M* cuts *OB* at the point *N*.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that *ON* : *NB* = 2 : 1

(2)

(Total for question = 6 marks)

Q7.

Relative to a fixed origin, points P, Q and R have position vectors **p**, **q** and **r** respectively.

Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(Total for question = 3 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB\rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB\rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a) M1: Attempts subtraction but may omit brackets
A1: cao (allow column vector notation)

(b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)
A1ft: $\lvert AB\rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Q2.

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\|AB\| = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\|AB\| = 3\sqrt{10}$	A1ft	1.1b
		(2)	
		(4 marks)	

Notes

(a)

M1: Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$

There must be some attempt to write in vector form.

A1: cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\mathrm{i} + 3\mathrm{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\mathrm{i} \\ 3\mathrm{j} \end{pmatrix}$

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $|AB| = \sqrt{(9)^2 + (3)^2}$ is also correct.

Condone missing brackets in the expression $|AB| = \sqrt{-9^2 + (3)^2}$

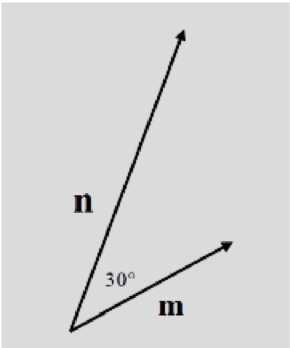
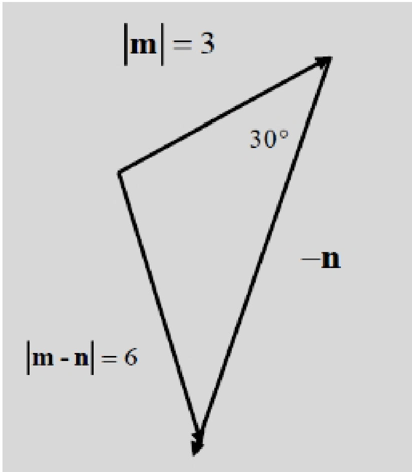
Also allow a restart usually accompanied by a diagram.

A1ft: $|AB| = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

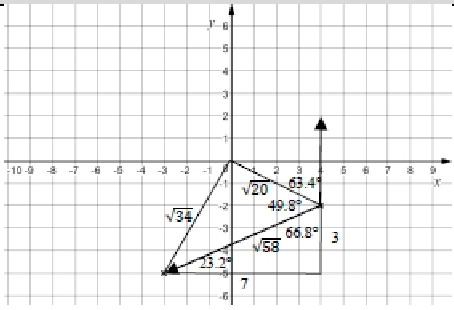
Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question

Q3.

Question	Scheme	Marks	AOs
(i)	Explains that $\mathbf{a}$ and $\mathbf{b}$ lie in the same direction oe	B1	2.4
		(1)	
(ii)	 	M1	1.1b
	Attempts $\dfrac{\sin 30°}{6} = \dfrac{\sin\theta}{3}$	M1	3.1a
	$\theta = \text{awrt } 14.5°$	A1	1.1b
	Angle between vector $\mathbf{m}$ and vector $\mathbf{m} - \mathbf{n}$ is awrt $135.5°$	A1	3.2a
		(4)	
			(5 marks)

Notes
(i) **B1:** Accept any valid response E.g The lines are collinear. Condone "They are parallel" Mark positively. ISW after a correct answer Do not accept "the length of line a +b is the same as the length of line a + the length of line b Do not accept $\|\mathbf{a}\|$ and $\|\mathbf{b}\|$ are parallel. **(ii)** **M1:** A triangle showing 3, 6 and $30°$ in the correct positions. Look for 6' opposite $30°$ with another side of 3. Condone the triangle not being obtuse angled and not being to scale. Do not condone negative lengths in the tringle. This would automatically be M0 **M1:** Correct sine rule statement with the sides and angles in the correct positions. If a triangle is drawn then the angles and sides must be in the correct positions. This is not dependent so allow recovery from negative lengths in the triangle. If the candidate has not drawn a diagram then correct sine rule would be M1 M1 Do not accept calculations where the sides have a negative length. Eg $\frac{\sin 30°}{6} = \frac{\sin\theta}{-3}$ is M0 **A1:** $\theta =$ awrt $14.5°$ **A1:** CSO awrt $135.5°$

Q4.

Question	Scheme	Marks	AOs
(a)			
	Attempts to find an "allowable" angle Eg $\tan\theta = \frac{7}{3}$	M1	1.1b
	A full attempt to find the bearing Eg $180° + "67°"$	dM1	3.1b
	Bearing = awrt $246.8°$	A1	1.1b
		(3)	
(b)	Attempts to find the distance travelled = $\sqrt{(4--3)^2+(-2+5)^2} = (\sqrt{58})$	M1	1.1b
	Attempts to find the speed = $\frac{\sqrt{58}}{2.75}$	dM1	3.1b
	= awrt 2.77 km h^{-1}	A1	1.1b
		(3)	
			(6 marks)

Notes: Score these two parts together.

(a)

M1: Attempts an allowable angle. (Either the "66.8", "23.2" or ("49.8" and "63.4"))

$\tan\theta = \pm\frac{7}{3}$, $\tan\theta = \pm\frac{3}{7}$, $\tan\theta = \pm\frac{-2--5}{4--3}$ etc

There must be an attempt to subtract the coordinates (seen or applied at least once)

If part (b) is attempted first, look for example for $\sin\theta = \pm\frac{7}{"\sqrt{58}"}$, $\cos\theta = \pm\frac{7}{"\sqrt{58}"}$, etc

They may use the cosine rule and trigonometry to find the two angles in the scheme. See above. Eg award for $\cos\theta = \frac{"58"+"20"-"34"}{2\times"\sqrt{58}"\times"\sqrt{20}"}$ and $\tan\theta = \pm\frac{4}{2}$ or equivalent.

dM1: A full attempt to find the bearing. $180° + \arctan\frac{7}{3}$, $270° - \arctan\frac{3}{7}$, $360° - "49.8°" - "63.4°"$. It is dependent on the previous method mark.

A1: Bearing = awrt $246.8°$ oe. Allow S 66.8° W

(b)

M1: Attempts to find the distance travelled. Allow for $d^2 = (4 - -3)^2 + (-2+5)^2$
You may see this on a diagram and allow if they attempt to find the magnitude from their "resultant vector" found in part (a).

dM1: Attempts to find the speed. There must have been an attempt to find the distance using the coordinates and then divide it by 2.75. Alternatively they could find the speed in km min^{-1} and then multiply by 60

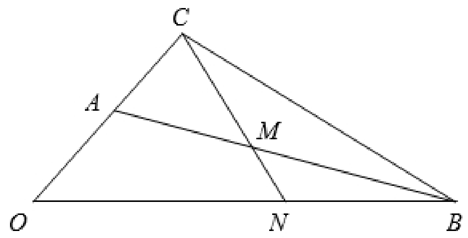
A1: awrt 2.77 km h^{-1}

Q5.

Question	Scheme	Marks	AOs
(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i} - 10\mathbf{j}) = -2 \times (12\mathbf{i} + 5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point O.	A1	2.4
		(2)	
(b)	Attempts distance $AB = \sqrt{(12+24)^2 + (10+5)^2}$	M1	1.1b
	Attempts speed = $\dfrac{\sqrt{(12+24)^2 + (10+5)^2}}{4}$	dM1	3.1a
	Speed = 9.75 ms^{-1}	A1	3.2a
		(3)	
			(5 marks)
Alt(a)	Attempts to find the equation of the line which passes through A and B E.g. $y - 5 = \dfrac{5+10}{12+24}(x-12)$ $\left(y = \dfrac{5}{12}x\right)$	M1	1.1b
	Shows that when $x = 0$, $y = 0$ and concludes the stone passes through the point O.	A1	2.4

Notes
(a) **M1:** Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ either way around. E.g. States that $(-24\mathbf{i}-10\mathbf{j}) = -2\times(12\mathbf{i}+5\mathbf{j})$ Alternatively, allow an attempt finding the gradient using any two of AO, OB or AB Alternatively attempts to find the equation of the line through A and B proceeding as far as $y = ...x$ Condone sign slips. **A1:** States that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ or as AO is parallel to OB (and the stone is travelling in a straight line) the stone passes through the point O. Alternatively, shows that the point $(0,0)$ is on the line and concludes (the stone) passes through the point O. **(b)** **M1:** Attempts to find the distance AB using a correct method. Condone slips but expect to see an attempt at $\sqrt{a^2+b^2}$ where a or b is correct **dM1:** Dependent upon the previous mark. Look for an attempt at $\frac{\text{distance } AB}{4}$ **A1:** $9.75\ \text{ms}^{-1}$ Requires units

Q6.

Part	Working or answer an examiner might expect to see	Mark	Notes
			
(a)	$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$	M1	This mark is given for a method to find an expression for $\overrightarrow{CM}$
	$\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{CM}$ in terms of **a** and **b**
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	This mark is given for a method to find an expression for $\overrightarrow{ON}$
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $= \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2}\lambda\right)\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{ON}$ in terms of **a** and **b**
(c)	$\left(2 - \frac{3}{2}\lambda\right) = 0$ so $\lambda = \frac{4}{3}$	M1	This mark is given for deducing that the coefficient of **a** = 0 and finding a value for λ
	$\overrightarrow{ON} = 0 \times \mathbf{a} + \frac{2}{3}\mathbf{b}$ Hence $ON{:}NB = \frac{2}{3}:\frac{1}{3} = 2{:}1$	A1	This mark is given for finding $\overrightarrow{ON}$ and giving a valid conclusion
			(Total 6 marks)

Q7.

Question Number	Scheme	Marks	AO's
	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$ $\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p}\Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})$ *	A1*	2.1
		(3)	
			(3 marks)

Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

A1*: Fully correct work leading to the given answer. Allow OQ = … as long as OQ has been defined as **q** earlier.

In the working allow use of P instead of **p** and Q instead of **q** as long as the intention is clear.