

Vectors Cheat Sheet

Vectors

A vector has both magnitude and direction.

You can represent a vector using a directed line segment. Direction of arrow shows the direction of vector



Example 1: The diagram shows vectors a, b and c. Draw a diagram to illustrate the vector addition





Subtracting a vector is same as adding a negative vector $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$



Multiplying a vector:

You can multiply a vector by a scalar. If the number is +ve the resultant vector has same direction but different length. If the number is -ve, the new vector has different length but opposite direction



Any vector parallel to the vector **a** may be written as λ **a** where λ is a non-zero scalar Parallelogram Law for vector addition: For the parallelogram ABCD, If $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{b}$ then $\overrightarrow{AC} = \mathbf{a} + \mathbf{b}$

Example 2: Show that the vectors 6a + 8b and 9a + 12b are parallel. Try to write one of the vectors as $\lambda \times$ other vector

 $9a + 12b = \frac{3}{2}(6a + 8b)$

Since, $9\mathbf{a} + 12\mathbf{b}$ can be written as $\lambda \times (6\mathbf{a} + 8\mathbf{b})$ where $\lambda = \frac{3}{2}$, this implies that $6\mathbf{a} + 8\mathbf{b}$ and $9\mathbf{a} + 12\mathbf{b}$ are parallel.

Representing vector:

A vector represent change in position or displacement relative to the change in x-and y –axes





To multiply a column vector by a scalar multiply each component by the scalar: $\lambda \begin{pmatrix} p \\ a \end{pmatrix} = \begin{pmatrix} \lambda p \\ \lambda a \end{pmatrix}$

To add 2 column vector add the x- component and the y-components: $\binom{p}{q} + \binom{r}{s} = \binom{p+r}{q+s}$

Example 3: Given that **a**=2**i**+3**j** and **b**=4**i**-**j**, find the vector 4**b**-**a** in terms of **i** and **j** Start by substituting vectors **a** and **b** in 4**b**-**a** and then simplify 4b-a = 4(4i - j) - (2i + 3j) $= (16\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) = (16 - 2)\mathbf{i} + (-4 - 3)\mathbf{j} = 14\mathbf{i} - 7\mathbf{j}$ Hence, $4\mathbf{b} - \mathbf{a}$ in terms of \mathbf{i} and \mathbf{j} is $14\mathbf{i} - 7\mathbf{j}$

Example 4: Given that $\mathbf{a} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ -2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$ find $2\mathbf{a} + \mathbf{b} + \mathbf{c}$ Start by substituting vectors **a**, **b** and **c** in 2**a** + **b** + **c**

$2\mathbf{a} + \mathbf{b} + \mathbf{c} = 2\binom{9}{7} + \binom{11}{-3} + \binom{-8}{-1} = \binom{18}{14} + \binom{11}{-3} + \binom{-8}{-1} = \binom{21}{10}$

Unit Vector:

A unit vector is a vector of length 1. The unit vectors along the x- and y-axes are usually denoted by i and j respectively. $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

You can write any two-dimensional vector in the form $\binom{p}{q} = p\mathbf{i}+q\mathbf{j}$

In this topic you will learn to find a unit vector in the direction of a given vector A unit vector in the direction of **a** is $\frac{a}{|a|}$

so if $|\mathbf{a}| = 5$, then a unit vector in the direction of **a** is $\frac{a}{r}$

Magnitude and direction:

To calculate the magnitude of vector you can use Pythagoras theorem. For the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} = \begin{pmatrix} x \\ y \end{pmatrix}$ the magnitude of the vector is given by $|a| = \sqrt{x^2 + y^2}$ The two straight lines on either side of the vector is the notation for magnitude of a vector

Example 4: **a** = 2**i** + 3**j**, **b** = 3**i** -4**j**. Find the exact value of the magnitude **a** + **b** Since we need to calculate exact value of **a** + **b**, first we will have to add the two vectors. We can do that by representing the vectors in column vector form and then add So, $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ implies $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 2+3 \\ 3-4 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ Use the formula for magnitude which is $|\mathbf{a}| = \sqrt{x^2 + y^2}$ Hence, $|\mathbf{a} + \mathbf{b}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$ Keep your answer in surd form as you are asked to find exact value.

Example 5: For vector $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$, find the unit vector in the same direction. A unit vector in the direction of **a** is $\frac{a}{a}$

Let us first calculate $|\mathbf{a}|$, so $|\mathbf{a}| = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

To find the unit vector in the direction of **a** first write the vector in column vector form and multiply by $\frac{1}{|a|}$

$$\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a}}{13} = \frac{1}{13} \begin{pmatrix} 5\\ -12 \end{pmatrix} = \begin{pmatrix} \frac{5}{13}\\ \frac{-12}{13} \end{pmatrix}$$

Hence, the unit vector in the direction of **a** is $\begin{pmatrix} 13\\ -12 \end{pmatrix}$

Example 6: Find the angle that the vector 3i - 5j makes with j axis



The angle the vector $3\mathbf{i} - 5\mathbf{j}$ makes with \mathbf{j} is 90° + angle θ Vector 3i - 5j on axes will be 3 units to the right on i axis and 5 units down on i axis

We can find angle θ by using trigonometric ratio $\tan \theta = \frac{opposite}{adjacent} = \frac{5}{3}$

So, $\tan \theta = \frac{5}{2}$ taking inverse we get $\theta = \tan^{-1}\left(\frac{5}{2}\right) = 59^{\circ}$ So the angle 3i - 5j makes with j is $90^\circ + 59^\circ = 149^\circ$

Position Vector:

Position vector are the vectors that gives us the position of a point relative to fixed origin In general, a point P with coordinates (p,q) has a position vector $\overrightarrow{OP} = p\mathbf{i} + q\mathbf{j} = \begin{pmatrix} p \\ q \end{pmatrix}$

Example7:
$$OA = 5i-2j$$
 and $AB = 3i$.
Start by drawing the vectors on the a
 \overrightarrow{OB} . It is easier to work with column v
in column vector form
 $\overrightarrow{OA} = {5 \choose -2}$ and $\overrightarrow{AB} = {3 \choose 4}$
Using triangle law of vector addition
 $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
 $\overrightarrow{OB} = {5 \choose -2} + {3 \choose 4} = {5+3 \choose -2+4} = {4 \choose 2}$
In i, j form $\overrightarrow{OB} = 8i+2j$

Solving geometric problem:

$$\overrightarrow{OP} = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu} \overrightarrow{AB} \\ = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu} \overrightarrow{OB} - \overrightarrow{OA}$$

(0, 1)

i (1, 0) x

in to the equation gives us,



Modelling with vectors

In mechanics, vectors have both magnitude and direction. For example velocity, displacement, and force. Magnitude of a vector is a scalar quantity - it has size but no direction

Vector quantity	Its mag
velocity	speed
displacement	distand

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position vector of B relative to O ii. the bearing of B from O
i) To find the position vector of B relat
We will have to represent the girl's journey on a diagram
\overrightarrow{OB} = pi + qi
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2km due east implies p= 2 and 3km due south
implies q = -3, you can refer to the diagram here
So \overrightarrow{OB} = (2\mathbf{i} - 3\mathbf{j}) \,\mathrm{km}
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line segment OB which is 90^{\circ} + angle \theta
 Using \tan \theta we can find angle \theta
  \tan \theta = \frac{3}{2}
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You are familiar with the formula for distance, Distance = speed \times time and substitute speed and time = 10 seconds in the formula. Use the magnitude formula to calculate speed Speed = $|5i-j| = \sqrt{5^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26} = 5.099$ Hence, Distance = $5.099 \times 10 = 51.0$ m

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+ 4**j.** Find the position vector of *B* axes to find position vector of B. From the diagram position vector of B is vector vectors, so write the vectors

vecto



You can use vector to find the position vector of a point that divides a line segment in a given ratio If the point *P* divides the line segment *AB* in the ratio λ : μ , then



Example 8: In the diagram the points A and B have position vectors a and b respectively

The point *P* divides *AB* in the ratio 1:2. Find the position vector of *P*

The position vector of P is vector \overrightarrow{OP} . You can refer to the figure above. To find \overrightarrow{OP} we can make use of the equation given above for **point** P **dividing the line segment** AB **in the ratio** λ : μ . Here $\lambda = 1$ and $\mu = 2$ and substituting them

 $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{2} (\overrightarrow{OB} - \overrightarrow{OA})$ You will have to write \overrightarrow{AB} in terms of the position vectors of A and B in order to find \overrightarrow{OP}

$$\overrightarrow{OB} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

If a and b are two non-parallel vectors and pa + qb = ra + sb then p = r and q = s

nitude

Example 9: A girl walks 2km due east from fixed point O to A, and then 3km due south from A to B. Find: i. The

itive	to	0,	



ii) To find bearing of B from O, you will have to calculate angle clockwise from North of O to

So the bearing of B from O is $90^\circ + 56.3^\circ = 146.3^\circ = 146^\circ$ Example 10: Find the distance moved by particle which travels for 10 seconds at velocity (5i-j) ms⁻¹ In order to calculate distance we need to find speed (i.e. magnitude of velocity)

