Trigonometry — *Answers*

1 **a** $\dfrac{2}{\cos x} = \dfrac{3}{\sin x}$

$\dfrac{\sin x}{\cos x} = \frac{3}{2}$

$\tan x = \frac{3}{2}$

$x = 56.3, 56.3 - 180$

$x = -123.7°, 56.3°$

b $\cot^2 \theta - \cot \theta + 1 + \cot^2 \theta = 4$

$2 \cot^2 \theta - \cot \theta - 3 = 0$

$(2 \cot \theta - 3)(\cot \theta + 1) = 0$

$\cot \theta = -1$ or $\frac{3}{2}$

$\tan \theta = -1$ or $\frac{2}{3}$

$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$ or $0.5880, \pi + 0.5880$

$\theta = 0.59$ (2dp), $\frac{3\pi}{4}$, 3.73 (2dp), $\frac{7\pi}{4}$

2 $2 \sin \theta \cos 30 + 2 \cos \theta \sin 30$

$= \sin \theta \cos 30 - \cos \theta \sin 30$

$\sqrt{3} \sin \theta + \cos \theta = \dfrac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta$

$\dfrac{\sqrt{3}}{2} \sin \theta = -\frac{3}{2} \cos \theta$

$\dfrac{\sin \theta}{\cos \theta} = -\sqrt{3}$

$\tan \theta = -\sqrt{3}$

$\theta = 180 - 60, 360 - 60$

$\theta = 120°, 300°$

3 **a** **i** $\operatorname{cosec} A = \dfrac{1}{2 - \sqrt{3}} \times \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$= \dfrac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$

ii $\operatorname{cosec}^2 A = (2 + \sqrt{3})^2$

$= 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$

$\cot^2 A = \operatorname{cosec}^2 A - 1 = 6 + 4\sqrt{3}$

b $3(1 - 2 \sin^2 x) - 8 \sin x + 5 = 0$

$3 \sin^2 x + 4 \sin x - 4 = 0$

$(3 \sin x - 2)(\sin x + 2) = 0$

$\sin x = \frac{2}{3}$ or -2 [no solutions]

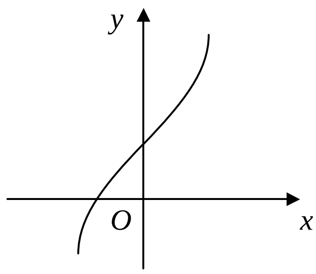
$x = 41.8, 180 - 41.8$

$x = 41.8°, 138.2°$

4 **a** $= \frac{\pi}{2} + 2 \times \frac{\pi}{6} = \frac{5\pi}{6}$

b $-\frac{\pi}{2} \le f(x) \le \frac{3\pi}{2}$

c



d $\frac{\pi}{2} + 2 \arcsin x = 0$

$\arcsin x = -\frac{\pi}{4}$

$x = \sin(-\frac{\pi}{4}) = -\dfrac{1}{\sqrt{2}}$

5 **a** $2\sin x - 3\cos x$

$= R\sin x\cos\alpha - R\cos x\sin\alpha$

$\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = 3$

$\therefore R = \sqrt{4+9} = \sqrt{13} = 3.61$

$\tan\alpha = \frac{3}{2},\ \alpha = 0.983$

$\therefore 2\sin x - 3\cos x = 3.61\sin(x - 0.983)$

b min. value = –3.61 (3sf)

when $x - 0.9828 = \frac{3\pi}{2},\ x = 5.70$ (3sf)

c $\sqrt{13}\sin(2x - 0.9828) + 1 = 0$

$\sin(2x - 0.9828) = -\frac{1}{\sqrt{13}}$

$2x - 0.983 = \pi + 0.2810, -0.2810$

$= -0.2810, 3.4226$

$2x = 0.7018, 4.4054$

$x = 0.35, 2.20$

6 **a** $\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$

let $A = B = \frac{x}{2}$

$\cos x \equiv \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$\cos x \equiv \cos^2\frac{x}{2} - (1 - \cos^2\frac{x}{2})$

$\cos x \equiv 2\cos^2\frac{x}{2} - 1$

b $\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1 + (2\cos^2\frac{x}{2} - 1)} = 3\cot\frac{x}{2}$

$\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = 3\cot\frac{x}{2}$

$\tan\frac{x}{2} = \frac{3}{\tan\frac{x}{2}}$

$\tan^2\frac{x}{2} = 3$

$\tan\frac{x}{2} = \pm\sqrt{3}$

$\frac{x}{2} = 60$ or $180 - 60$

$\frac{x}{2} = 60, 120$

$x = 120°, 240°$

7 **a** LHS $= \frac{1}{\sin\theta} - \sin\theta$

$= \frac{1 - \sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$

$= \cos\theta \times \frac{\cos\theta}{\sin\theta}$

$= \cos\theta\cot\theta$

= RHS

b $\frac{2}{\cos x} + \frac{\sin x}{\cos x} = 2\cos x$

$2 + \sin x = 2\cos^2 x$

$2 + \sin x = 2(1 - \sin^2 x)$

$2\sin^2 x + \sin x = 0$

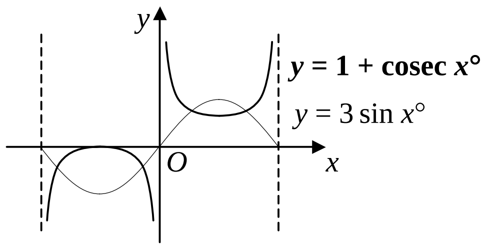
$\sin x(2\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or 0

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ or $0, \pi, 2\pi$

$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

8 **a**



b $3\sin x = 1 + \frac{1}{\sin x}$

$3\sin^2 x - \sin x - 1 = 0$

$\sin x = \frac{1 \pm \sqrt{1+12}}{6} = \frac{1 \pm \sqrt{13}}{6}$

$\sin x = -0.4343$ or 0.7676

$x = -25.7, 25.7 - 180$ or $50.1, 180 - 50.1$

$x = -154.3, -25.7, 50.1, 129.9$

9 **a** LHS = $\sec x + \tan x - \tan x - \sin x \tan x$

$= \frac{1}{\cos x} - \sin x \times \frac{\sin x}{\cos x}$

$= \frac{1 - \sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x}$

$= \cos x$

= RHS

b $2(1 + \tan^2 2y) + \tan^2 2y = 3$

$\tan^2 2y = \frac{1}{3}$

$\tan 2y = \pm\frac{1}{\sqrt{3}}$

$2y = \frac{\pi}{6}, \pi + \frac{\pi}{6}$ or $\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$y = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

10 **a** $4\sin x - \cos x$

$= R\sin x\cos\alpha - R\cos x\sin\alpha$

$\Rightarrow R\cos\alpha = 4,\ R\sin\alpha = 1$

$\therefore\ R = \sqrt{16+1} = \sqrt{17} = 4.12$

$\tan\alpha = \frac{1}{4},\ \alpha = 14.0$

$\therefore\ 4\sin x° - \cos x° = 4.12\sin(x - 14.0)°$

b $\frac{2}{\sin x} - \frac{\cos x}{\sin x} + 4 = 0$

$2 - \cos x + 4\sin x = 0$

$\therefore\ 4\sin x° - \cos x° + 2 = 0$

c $\sqrt{17}\sin(x - 14.04) + 2 = 0$

$\sin(x - 14.04) = -\frac{2}{\sqrt{17}}$

$x - 14.04 = 180 + 29.02, 360 - 29.02$

$= 209.02, 330.98$

$x = 223.1, 345.0$ (1dp)

11 **a** adding

$\cos(A+B) + \cos(A-B) \equiv 2\cos A\cos B$

let $P = A + B$ (1) and $Q = A - B$ (2)

(1) + (2) $\Rightarrow 2A = P + Q \Rightarrow A = \frac{P+Q}{2}$

(1) – (2) $\Rightarrow 2B = P - Q \Rightarrow B = \frac{P-Q}{2}$

$\therefore\ \cos P + \cos Q \equiv 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

b $2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2} + \cos 2x = 0$

$2\cos 2x\cos(-x) + \cos 2x = 0$

$\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$ or $\cos x = -\frac{1}{2}$

$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$

or $x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}$

12 **a** $3\cos\theta + 4\sin\theta$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$

$\therefore\ R = \sqrt{9+16} = 5$

$\tan\alpha = \frac{4}{3},\ \alpha = 0.927$ (3sf)

$\therefore\ 3\cos\theta + 4\sin\theta = 5\cos(\theta - 0.927)$

b **i** $-4 \le f(\theta) \le 6$

ii $1 - 5\cos(2\theta - 0.9273) = 0$

$\cos(2\theta - 0.9273) = \frac{1}{5}$

$2\theta - 0.9273 = 1.3694, 2\pi - 1.3694$

$= 1.3694, 4.9137$

$2\theta = 2.2967, 5.8410$

$\theta = 1.15, 2.92$ (2dp)

c $y = \frac{2}{5\cos(x - 0.9273)}$

TP: $y = \frac{2}{5}$ when $x - 0.9273 = 0$

$y = -\frac{2}{5}$ when $x - 0.9273 = \pi$

$\therefore\ (0.93, \frac{2}{5})$ and $(4.07, -\frac{2}{5})$