TRIGONOMETRY

1 **a** Solve the equation

$$2 \sec x - 3 \operatorname{cosec} x = 0,$$

for x in the interval $-180° \leq x \leq 180°$. **(4)**

b Find all values of θ in the interval $0 \leq \theta \leq 2\pi$ for which

$$\cot^2 \theta - \cot \theta + \operatorname{cosec}^2 \theta = 4.$$ **(6)**

2 For values of θ in the interval $0 \leq \theta \leq 360°$, solve the equation

$$2 \sin (\theta + 30°) = \sin (\theta - 30°).$$ **(6)**

3 **a** Given that $\sin A = 2 - \sqrt{3}$, find in the form $a + b\sqrt{3}$ the exact value of

i $\operatorname{cosec} A$,

ii $\cot^2 A$. **(5)**

b Solve the equation

$$3 \cos 2x - 8 \sin x + 5 = 0,$$

for values of x in the interval $0 \leq x \leq 360°$, giving your answers to 1 decimal place. **(5)**

4 $$\mathrm{f} : x \to \tfrac{\pi}{2} + 2 \arcsin x,\ x \in \mathbb{R},\ -1 \leq x \leq 1.$$

a Find the exact value of $\mathrm{f}(\tfrac{1}{2})$. **(2)**

b State the range of f. **(2)**

c Sketch the curve $y = \mathrm{f}(x)$. **(2)**

d Solve the equation $\mathrm{f}(x) = 0$. **(3)**

5 **a** Express $2 \sin x - 3 \cos x$ in the form $R \sin (x - \alpha)$, where $R > 0$ and $0 < \alpha < \tfrac{\pi}{2}$.

Give the values of R and α to 3 significant figures. **(4)**

b State the minimum value of $2 \sin x - 3 \cos x$ and the smallest positive value of x for which this minimum occurs. **(3)**

c Solve the equation

$$2 \sin 2x - 3 \cos 2x + 1 = 0,$$

for x in the interval $0 \leq x \leq \pi$, giving your answers to 2 decimal places. **(5)**

6 **a** Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 2 \cos^2 \frac{x}{2} - 1.$$ **(3)**

b Solve the equation

$$\frac{\sin x}{1 + \cos x} = 3 \cot \frac{x}{2},$$

for values of x in the interval $0 \leq x \leq 360°$. **(7)**

7 **a** Prove the identity

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta, \quad \theta \neq n\pi,\ n \in \mathbb{Z}.$$ **(3)**

b Find the values of x in the interval $0 \le x \le 2\pi$ for which

$$2\sec x + \tan x = 2\cos x,$$

giving your answers in terms of π. **(6)**

8 **a** Sketch on the same diagram the curves $y = 3\sin x°$ and $y = 1 + \operatorname{cosec} x°$ for x in the interval $-180 \le x \le 180$. **(4)**

b Find the x-coordinate of each point where the curves intersect in this interval, giving your answers correct to 1 decimal place. **(6)**

9 **a** Prove the identity

$$(1 - \sin x)(\sec x + \tan x) \equiv \cos x, \quad x \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}.$$ **(4)**

b Find the values of y in the interval $0 \le y \le \pi$ for which

$$2\sec^2 2y + \tan^2 2y = 3,$$

giving your answers in terms of π. **(6)**

10 **a** Express $4\sin x° - \cos x°$ in the form $R\sin(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.

Give the values of R and α to 3 significant figures. **(4)**

b Show that the equation

$$2\operatorname{cosec} x° - \cot x° + 4 = 0 \qquad \text{(I)}$$

can be written in the form

$$4\sin x° - \cos x° + 2 = 0.$$ **(2)**

c Using your answers to parts **a** and **b**, solve equation (I) for x in the interval $0 \le x \le 360$. **(4)**

11 **a** Use the identities

$$\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$$

and

$$\cos(A - B) \equiv \cos A\cos B + \sin A\sin B$$

to prove that

$$\cos P + \cos Q \equiv 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}.$$ **(4)**

b Find, in terms of π, the values of x in the interval $0 \le x \le 2\pi$ for which

$$\cos x + \cos 2x + \cos 3x = 0.$$ **(7)**

12 **a** Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

b Given that the function f is defined by

$$f(\theta) \equiv 1 - 3\cos 2\theta - 4\sin 2\theta,\ \theta \in \mathbb{R},\ 0 \le \theta \le \pi,$$

i state the range of f,

ii solve the equation $f(\theta) = 0$. **(6)**

c Find the coordinates of the turning points of the curve with equation $y = \dfrac{2}{3\cos x + 4\sin x}$ in the interval $[0, 2\pi]$. **(3)**