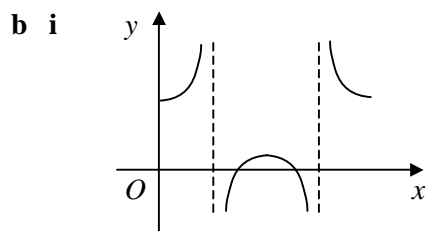## TRIGONOMETRY                                    *Answers*

**1**   $\sec^2 x - 1 - \sec x = 1$
$\sec^2 x - \sec x - 2 = 0$
$(\sec x + 1)(\sec x - 2) = 0$
$\sec x = -1$ or $2$
$\cos x = -1$ or $\frac{1}{2}$
$x = 180$ or $60, 360 - 60$
$x = 60°, 180°, 300°$

**2**   **a**   $2 \cos x + 5 \sin x$
$\qquad = R \cos x \cos \alpha + R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 2, \ R \sin \alpha = 5$
$\therefore \ R = \sqrt{4 + 25} \ = \sqrt{29} \ = 5.39$
$\quad \tan \alpha = \frac{5}{2}, \ \alpha = 68.2$
$\therefore \ 2 \cos x + 5 \sin x = 5.39 \cos (x - 68.2)°$

**b**   $\sqrt{29} \cos (x - 68.199) = 3$
$\cos (x - 68.199) = \dfrac{3}{\sqrt{29}} = 0.5571$
$x - 68.199 = 56.145, -56.145$
$x = 12.1, 124.3$

**3**   **a**   $\arctan 2x = \frac{\pi}{6}$
$2x = \tan \frac{\pi}{6} = \dfrac{1}{\sqrt{3}}$
$x = \dfrac{1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

**b**   $4 \sin x \cos x = 3 \cos x$
$\cos x(4 \sin x - 3) = 0$
$\cos x = 0$ or $\sin x = \frac{3}{4}$
$x = 90, 360 - 90$ or $48.6, 180 - 48.6$
$x = 48.6° \ (1\mathrm{dp}), 90°, 131.4° \ (1\mathrm{dp}), 270°$

**4**   **a**   $\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$
$\sin (A - B) \equiv \sin A \cos B - \cos A \sin B$

subtracting

$\sin (A + B) - \sin (A + B) \equiv 2 \cos A \sin B$
let $P = A + B$ (1) and $Q = A - B$ (2)
$(1) + (2) \ \Rightarrow \ 2A = P + Q \ \Rightarrow \ A = \dfrac{P + Q}{2}$
$(1) - (2) \ \Rightarrow \ 2B = P - Q \ \Rightarrow \ B = \dfrac{P - Q}{2}$
$\therefore \ \sin P - \sin Q \equiv 2 \cos \dfrac{P + Q}{2} \sin \dfrac{P - Q}{2}$

**b**   $\sin 4x - \sin 2x = 0$
$2 \cos \frac{4x + 2x}{2} \sin \frac{4x - 2x}{2} = 0$
$\cos 3x \sin x = 0$
$\cos 3x = 0$ or $\sin x = 0$
$3x = 90, 360 - 90, 360 + 90$ or $x = 0, 180$
$3x = 90, 270, 450$ or $x = 0, 180$
$x = 0, 30°, 90°, 150°, 180°$

**5**   **a**   $\text{LHS} = 4\sin^2\theta - 4 + \csc^2\theta$

   $= 4(1 - \cos^2\theta) - 4 + \csc^2\theta$

   $= \csc^2\theta - 4\cos^2\theta = \text{RHS}$

   **b**   **i**



   **ii**   $(0, 5)$

   **iii**   $3 + 2\sec x = 0$

   $\sec x = -\tfrac{3}{2}, \ \cos x = -\tfrac{2}{3}$

   $x = \pi - 0.841, \ \pi + 0.841 = 2.30, 3.98$

   $\therefore \ (2.30, 0) \text{ and } (3.98, 0) \ [x \text{ to 2dp}]$

**6**   **a**   $\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

   $\Rightarrow R\cos\alpha = 1, \ R\sin\alpha = 1$

   $\therefore \ R = \sqrt{1+1} = \sqrt{2}$

   $\tan\alpha = 1, \ \alpha = \tfrac{\pi}{4}$

   **b**   $\cos x - \sin x + \sqrt{2}\cos(3x - \tfrac{\pi}{4}) = 0$

   $\sqrt{2}\cos(x + \tfrac{\pi}{4}) + \sqrt{2}\cos(3x - \tfrac{\pi}{4}) = 0$

   $2\sqrt{2}\cos\tfrac{4x}{2}\cos\tfrac{-2x+\frac{\pi}{2}}{2} = 0$

   $\cos 2x\cos(\tfrac{\pi}{4} - x) = 0$

   $\cos 2x\cos(x - \tfrac{\pi}{4}) = 0$

   $\cos 2x = 0 \ \text{ or } \ \cos(x - \tfrac{\pi}{4}) = 0$

   $2x = \tfrac{\pi}{2}, 2\pi - \tfrac{\pi}{2}, 2\pi + \tfrac{\pi}{2}, 4\pi - \tfrac{\pi}{2}$

   $\text{or } \ x - \tfrac{\pi}{4} = \tfrac{\pi}{2}, 2\pi - \tfrac{\pi}{2}$

   $2x = \tfrac{\pi}{2}, \tfrac{3\pi}{2}, \tfrac{5\pi}{2}, \tfrac{7\pi}{2} \ \text{ or } \ x - \tfrac{\pi}{4} = \tfrac{\pi}{2}, \tfrac{3\pi}{2}$

   $x = \tfrac{\pi}{4}, \tfrac{3\pi}{4}, \tfrac{5\pi}{4}, \tfrac{7\pi}{4}$

**7**   **a**   $\text{LHS} = \dfrac{\cos 2x}{\sin 2x} + \dfrac{1}{\sin 2x}$

   $= \dfrac{\cos 2x + 1}{\sin 2x}$

   $= \dfrac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$

   $= \dfrac{2\cos^2 x}{2\sin x\cos x}$

   $= \dfrac{\cos x}{\sin x}$

   $= \cot x = \text{RHS}$

   **b**   $\cot x = 6 - \cot^2 x$

   $\cot^2 x + \cot x - 6 = 0$

   $(\cot x + 3)(\cot x - 2) = 0$

   $\cot x = -3 \ \text{ or } \ 2$

   $\tan x = -\tfrac{1}{3} \ \text{ or } \ \tfrac{1}{2}$

   $x = \pi - 0.3218, 2\pi - 0.3218$

   $\text{or } \ 0.4636, \pi + 0.4636$

   $x = 0.46, 2.82, 3.61, 5.96$

**8**   **a**   $\text{LHS} = \cos x\cos 30 - \sin x\sin 30 + \sin x$

   $= \dfrac{\sqrt{3}}{2}\cos x - \tfrac{1}{2}\sin x + \sin x$

   $= \dfrac{\sqrt{3}}{2}\cos x + \tfrac{1}{2}\sin x$

   $= \cos x\cos 30 + \sin x\sin 30$

   $= \cos(x - 30)° = \text{RHS}$

   **b**   let $x = 45$

   $\cos 75° + \sin 45° = \cos 15°$

   $\therefore \ \cos 75° - \cos 15° = -\sin 45°$

   $= -\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = -\tfrac{1}{2}\sqrt{2}$

   **c**   $3\cos(x + 30) + 3\sin x - 2\sin x$

   $= 3\cos(x - 30) + 1$

   $-2\sin x = 1$

   $\sin x = -\tfrac{1}{2}$

   $x = -30, 30 - 180$

   $x = -150, -30$

**9**    **a**   $a = 3$

     $b \sin x° + c \cos x°$ can be expressed in

     the form $k \sin (x + \alpha)°$ which will vary

     between $-k$ and $+k$

     $\therefore a + k = 5$ and $a - k = 1$, hence $a = 3$

   **b**   $3 + k = 5 \therefore k = 2$

     $60 + \alpha = 90 \therefore \alpha = 30$

   **c**   $f(x) = 3 + 2 \sin (x + 30)$

       $= 3 + 2 \sin x \cos 30 + 2 \cos x \sin 30$

       $= 3 + \sqrt{3} \sin x + \cos x$

     $\therefore b = \sqrt{3}, \; c = 1$

**10**   **a**   LHS $= \dfrac{1 - (1 - 2\sin^2 \frac{x}{2})}{1 + (2\cos^2 \frac{x}{2} - 1)}$

        $= \dfrac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$

        $= \tan^2 \dfrac{x}{2} = $ RHS

   **b**   **i**   let $x = \frac{\pi}{6}$, $\dfrac{1 - \cos \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}} = \tan^2 \frac{\pi}{12}$

         $\tan^2 \frac{\pi}{12} = \dfrac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} = \dfrac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \dfrac{2 - \sqrt{3}}{2 - \sqrt{3}}$

            $= \dfrac{4 - 4\sqrt{3} + 3}{4 - 3} = 7 - 4\sqrt{3}$

     **ii**   $\tan^2 \frac{x}{2} = 1 - \sec \frac{x}{2}$

        $\sec^2 \frac{x}{2} - 1 = 1 - \sec \frac{x}{2}$

        $\sec^2 \frac{x}{2} + \sec \frac{x}{2} - 2 = 0$

        $(\sec \frac{x}{2} + 2)(\sec \frac{x}{2} - 1) = 0$

        $\sec \frac{x}{2} = -2$ or $1$

        $\cos \frac{x}{2} = -\frac{1}{2}$ or $1$

        $\frac{x}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or $0$

        $x = 0, \frac{4\pi}{3}$

**11**   **a**   $6\cot^2 x - \text{cosec } x + 5 = 0$

       $\Rightarrow \; 6(\text{cosec}^2 x - 1) - \text{cosec } x + 5 = 0$

       $6\text{cosec}^2 x - \text{cosec } x - 1 = 0$

       $(3 \text{ cosec } x + 1)(2 \text{ cosec } x - 1) = 0$

       $\text{cosec } x = -\frac{1}{3}, \frac{1}{2}$

     for real $x$, $|\text{cosec } x| \geq 1$

     $\therefore$ no real solutions

   **b**   $\cos 5y - \cos y = 0$

     $-2 \sin \frac{5y+y}{2} \sin \frac{5y-y}{2} = 0$

     $\sin 3y \sin 2y = 0$

     $\sin 3y = 0$ or $\sin 2y = 0$

     $3y = 0, 180, 360, 540$ or $2y = 0, 180, 360$

     $y = 0, 60°, 90°, 120°, 180°$

**12**   **a**   $\cos (A - B) \equiv \cos A \cos B + \sin A \sin B$

     $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$

     subtracting

     $\cos (A - B) - \cos (A + B) \equiv 2 \sin A \sin B$

     $\therefore \sin A \sin B \equiv \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

   **b**   $4 \sin (x + \frac{\pi}{3}) = \dfrac{1}{\sin(x - \frac{\pi}{6})}$

     $4 \sin (x + \frac{\pi}{3}) \sin (x - \frac{\pi}{6}) = 1$

     $2[\cos \frac{\pi}{2} - \cos (2x + \frac{\pi}{6})] = 1$

     $2[0 - \cos (2x + \frac{\pi}{6})] = 1$

     $\cos (2x + \frac{\pi}{6}) = -\frac{1}{2}$

     $2x + \frac{\pi}{6} = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

          $= \frac{2\pi}{3}, \frac{4\pi}{3}$

     $2x = \frac{\pi}{2}, \frac{7\pi}{6}$

     $x = \frac{\pi}{4}, \frac{7\pi}{12}$