

TRIGONOMETRY

Answers

1 a $x + 40 = \pm 72.5$

$$x = -112.5^\circ, 32.5^\circ$$

b $\tan 2x = -2$

$$2x = 180 - 63.435, 360 - 63.435,$$

$$-63.435, -180 - 63.435$$

$$= -243.435, -63.435, 116.565, 296.565$$

$$x = -121.7^\circ, -31.7^\circ, 58.3^\circ, 148.3^\circ$$

2 $\tan x = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \frac{1}{2}\sqrt{2}$

$$x = 59.6, 180 + 59.6 \text{ or } 16.3, 180 + 16.3$$

$$x = 16.3, 59.6, 196.3, 239.6$$

3 a $15\theta = 32.1$

$$\theta = 32.1 \div 15 = 2.14$$

b $A = \frac{1}{2} \times 15^2 \times 2.14$

$$= 240.75 \text{ cm}^2$$

4 $2x - \frac{\pi}{3} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2x = \frac{\pi}{2}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12}$$

5 a $\sin^2 A = (1 - \sqrt{2})^2$

$$= 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$$

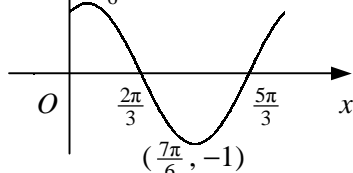
$$\cos^2 A = 1 - \sin^2 A = 2\sqrt{2} - 2$$

$$\therefore \cos^2 A + 2 \sin A$$

$$= 2\sqrt{2} - 2 + 2(1 - \sqrt{2})$$

$$\therefore \cos^2 A + 2 \sin A = 0$$

b y $(\frac{\pi}{6}, 1)$



7 a $\frac{\sin(\angle PRQ)}{10} = \frac{\sin 0.7}{14}$

$$\sin(\angle PRQ) = \frac{10 \times \sin 0.7}{14} = 0.4602$$

$$\angle PRQ = 0.48^\circ$$

b $\angle PQR = \pi - (0.7 + 0.4782) = 1.963$

$$\text{area of } \Delta = \frac{1}{2} \times 10 \times 14 \times \sin 1.963$$

$$= 64.67$$

$$\text{area of sector} = \frac{1}{2} \times 10^2 \times 0.7$$

$$= 35$$

$$\text{shaded area} = 64.67 - 35$$

$$= 29.7 \text{ cm}^2 \text{ (3sf)}$$

8 a i $\cos^2 A = 1 - \sin^2 A = 1 - \frac{5}{9} = \frac{4}{9}$

$$\cos A = \pm \sqrt{\frac{4}{9}} = \pm \frac{2}{3}$$

$$0 < A < 90 \therefore \cos A = \frac{2}{3}$$

ii $\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{5}}{3} \div \frac{2}{3} = \frac{1}{2}\sqrt{5}$

b $\cos x (5 \sin x + 1) = 0$

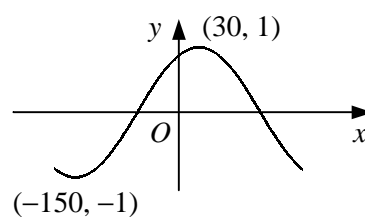
$$\cos x = 0 \text{ or } \sin x = -0.2$$

$$x = 90, 270 \text{ or } 180 + 11.5, 360 - 11.5$$

$$x = 90^\circ, 191.5^\circ \text{ (1dp), } 270^\circ, 348.5^\circ \text{ (1dp)}$$

$$\begin{aligned}
 9 \quad 2\theta + 30 &= 180 - 60, 180 + 60 \\
 &= 120, 240 \\
 2\theta &= 90, 210 \\
 \theta &= 45, 105
 \end{aligned}$$

10 a



$$\begin{aligned}
 \text{b} \quad \cos(x - 30) &= 0.2 \\
 x - 30 &= \pm 78.5 \\
 x &= -48.5, 108.5
 \end{aligned}$$

$$\begin{aligned}
 11 \quad 4 \cos^2 x - \cos x - 2(1 - \cos^2 x) &= 0 \\
 6 \cos^2 x - \cos x - 2 &= 0 \\
 (3 \cos x - 2)(2 \cos x + 1) &= 0 \\
 \cos x &= \frac{2}{3} \text{ or } -0.5 \\
 x &= 48.2, 360 - 48.2 \text{ or } 180 - 60, 180 + 60 \\
 x &= 48.2^\circ \text{ (1dp), } 120^\circ, 240^\circ, 311.8^\circ \text{ (1dp)}
 \end{aligned}$$

$$\begin{aligned}
 12 \quad \text{a} \quad \text{area of sector} &= \frac{1}{2} \times r^2 \times \theta \\
 \text{area of triangle} &= \frac{1}{2} \times r^2 \times \sin \theta \\
 A_1 &= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta \\
 &= \frac{1}{2} r^2 (\theta - \sin \theta) \text{ cm}^2 \\
 \text{b} \quad \theta &= \frac{5\pi}{6} \quad \therefore A_1 = \frac{1}{2} r^2 \left(\frac{5\pi}{6} - \frac{1}{2} \right) \\
 &= \frac{1}{12} r^2 (5\pi - 3) \\
 A_2 &= \pi r^2 - A_1 = \pi r^2 - \left(\frac{5}{12} \pi r^2 - \frac{1}{4} r^2 \right) \\
 &= \frac{7}{12} \pi r^2 + \frac{1}{4} r^2 \\
 &= \frac{1}{12} r^2 (7\pi + 3) \\
 \therefore A_1 : A_2 &= \frac{1}{12} r^2 (5\pi - 3) : \frac{1}{12} r^2 (7\pi + 3) \\
 &= (5\pi - 3) : (7\pi + 3)
 \end{aligned}$$

$$\begin{aligned}
 13 \quad 3 \sin x - 2 \cos^2 x &= 0 \\
 3 \sin x - 2(1 - \sin^2 x) &= 0 \\
 2 \sin^2 x + 3 \sin x - 2 &= 0 \\
 (2 \sin x - 1)(\sin x + 2) &= 0 \\
 \sin x &= 0.5 \text{ or } -2 \text{ [no solutions]} \\
 x &= \frac{\pi}{6}, \pi - \frac{\pi}{6} \\
 x &= \frac{\pi}{6}, \frac{5\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \text{a} \quad 7^2 &= 5^2 + 8^2 - [2 \times 5 \times 8 \times \cos(\angle ABC)] \\
 \cos(\angle ABC) &= \frac{25 + 64 - 49}{80} \\
 &= \frac{1}{2} \\
 \text{b} \quad \sin(\angle ABC) &= \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \\
 \text{area} &= \frac{1}{2} \times 5 \times 8 \times \frac{\sqrt{3}}{2} \\
 &= 10\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad \text{a} \quad \text{LHS} &= 2 + 2 \tan^2 \theta + \cos^2 \theta + \sin^2 \theta \\
 &= 2 + 2 \tan^2 \theta + 1 \\
 &= 3 + 2 \tan^2 \theta \\
 &= \text{RHS} \\
 \text{b} \quad 3 + 2 \tan^2 \theta &= 7 \\
 \tan^2 \theta &= 2 \\
 \tan \theta &= \pm \sqrt{2} \\
 \theta &= 54.7, 180 + 54.7 \\
 &\text{or } 180 - 54.7, 360 - 54.7 \\
 \theta &= 54.7^\circ, 125.3^\circ, 234.7^\circ, 305.3^\circ \text{ (1dp)}
 \end{aligned}$$