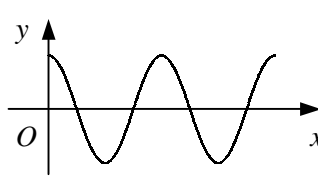


TRIGONOMETRY

Answers

- 1 a** $\tan x = \frac{1}{\sqrt{3}}$
 $x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$
 $x = \frac{\pi}{6}, \frac{7\pi}{6}$
- b** $\cos(x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$
 $x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$
 $= \frac{5\pi}{6}, \frac{7\pi}{6}$
 $x = \frac{\pi}{2}, \frac{5\pi}{6}$
- 2 a** $\cos^2 A = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$
 $\sin^2 A = 1 - \cos^2 A = 2\sqrt{3} - 3$
- b** $\tan^2 A = \frac{\sin^2 A}{\cos^2 A}$
 $= \frac{2\sqrt{3}-3}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{(2\sqrt{3}-3)(4+2\sqrt{3})}{16-12}$
 $= \frac{8\sqrt{3}+12-12-6\sqrt{3}}{4} = \frac{2\sqrt{3}}{4}$
 $= \frac{\sqrt{3}}{2}$
- 3 a** $45^\circ = \frac{\pi}{4}$
 $P = (2 \times 8) + (8 \times \frac{\pi}{4}) = 22.3 \text{ cm}$
- b** area of sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi$
area of triangle $= \frac{1}{2} \times 8^2 \times \sin \frac{\pi}{4}$
 $= 32 \times \frac{1}{\sqrt{2}} = 16\sqrt{2}$
area of segment $= 8\pi - 16\sqrt{2}$
 $= 8(\pi - 2\sqrt{2}) \text{ cm}^2$
- 4** $2 \sin^2 \theta + \sin \theta - (1 - \sin^2 \theta) = 2$
 $3 \sin^2 \theta + \sin \theta - 3 = 0$
 $\sin \theta = \frac{-1 \pm \sqrt{1+36}}{6}$
 $\sin \theta = \frac{1}{6}(-1 + \sqrt{37})$
or $\frac{1}{6}(-1 - \sqrt{37})$ [no solutions]
 $\theta = 57.9, 180 - 57.9$
 $\theta = 57.9^\circ, 122.1^\circ$
- 5** $3 \sin^2 x + 4 \sin x - 4 = 0$
 $(3 \sin x - 2)(\sin x + 2) = 0$
 $\sin x = \frac{2}{3}$ or -2 [no solutions]
 $x = 0.73, \pi - 0.7297$
 $x = 0.73^\circ, 2.41^\circ$
- 6** $2 \sin x = 3 \cos x$
 $\tan x = 1.5$
 $x = 0.98, \pi + 0.9828 = 0.98, 4.12$
 $\therefore (0.98, 1.66), (4.12, -1.66)$
- 7 a** 
- b** $2x = 180 - 60, 180 + 60,$
 $540 - 60, 540 + 60$
 $= 120, 240, 480, 600$
 $x = 60, 120, 240, 300$
- 8** $12 \cos^2 \theta = 7 \sin \theta$
 $12(1 - \sin^2 \theta) = 7 \sin \theta$
 $12 \sin^2 \theta + 7 \sin \theta - 12 = 0$
 $(4 \sin \theta - 3)(3 \sin \theta + 4) = 0$
 $\sin \theta = 0.75$ or $-\frac{4}{3}$ [no solutions]
 $\theta = 48.6, 180 - 48.6$
 $\theta = 48.6, 131.4$

$$\begin{aligned} 9 \quad \mathbf{a} \quad \tan 15^\circ &= \frac{\sqrt{3}-1}{1+(\sqrt{3}\times 1)} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} \\ &= \frac{(\sqrt{3}-1)(1-\sqrt{3})}{1-3} \\ &= -\frac{1}{2}(\sqrt{3}-3-1+\sqrt{3}) \\ &= 2 - \sqrt{3} \end{aligned}$$

$$\mathbf{b} \quad \tan 345^\circ = -\tan 15^\circ = \sqrt{3} - 2$$

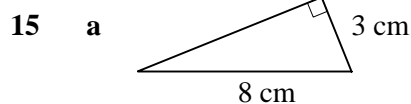
$$\begin{aligned} 11 \quad \mathbf{a} \quad \angle ABC &= 180 - (41 + 26) = 113 \\ \frac{BC}{\sin 41} &= \frac{18}{\sin 113} \\ BC &= \frac{18 \times \sin 41}{\sin 113} = 12.8 \text{ cm} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad &= \frac{1}{2} \times 18 \times 12.829 \times \sin 26 \\ &= 50.6 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 13 \quad \sin^2 x + 5 \sin x &= 2(1 - \sin^2 x) \\ 3 \sin^2 x + 5 \sin x - 2 &= 0 \\ (3 \sin x - 1)(\sin x + 2) &= 0 \\ \sin x &= \frac{1}{3} \quad \text{or} \quad -2 \text{ [no solutions]} \end{aligned}$$

$$x = 19.5, 180 - 19.5$$

$$x = 19.5^\circ, 160.5^\circ$$



$$\cos(\angle PQR) = \frac{3}{8}$$

$$\therefore \angle PQR = 1.186^\circ$$

$$\begin{aligned} \mathbf{b} \quad RS^2 &= 8^2 - 3^2 = 55 \\ RS &= \sqrt{55} = 7.42 \text{ cm (3sf)} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad \text{obtuse } \angle SPU &= 2 \times \angle PQR = 2.3728 \\ \text{reflex } \angle RQT &= 2\pi - \angle SPU = 3.9104 \\ \text{length of rubber band} \\ &= (2 \times 7.4162) + (2 \times 2.3728) + (5 \times 3.9104) \\ &= 39.1 \text{ cm (3sf)} \end{aligned}$$

$$10 \quad (1 - \cos^2 x) + 5 \cos x - 3 \cos^2 x = 2$$

$$4 \cos^2 x - 5 \cos x + 1 = 0$$

$$(4 \cos x - 1)(\cos x - 1) = 0$$

$$\cos x = 0.25 \quad \text{or} \quad 1$$

$$x = 75.5, 360 - 75.5 \quad \text{or} \quad 0, 360$$

$$x = 0, 75.5^\circ \text{ (1dp)}, 284.5^\circ \text{ (1dp)}, 360^\circ$$

$$12 \quad 6 \cos^2 \theta + 5 \cos \theta - 4 = 0$$

$$(3 \cos \theta + 4)(2 \cos \theta - 1) = 0$$

$$\cos \theta = 0.5 \quad \text{or} \quad -\frac{4}{3} \text{ [no solutions]}$$

$$\theta = 60, 360 - 60$$

$$\theta = 60^\circ, 300^\circ$$

$$\begin{aligned} 14 \quad \mathbf{a} \quad \text{LHS} &= (1 - \cos^2 \theta)^2 - 2(1 - \cos^2 \theta) \\ &= 1 - 2 \cos^2 \theta + \cos^4 \theta - 2 + 2 \cos^2 \theta \\ &= \cos^4 \theta - 1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \text{LHS} &= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)} \\ &= \frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)} \\ &= \frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} \\ &= \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} \\ &= \frac{2}{\sin \theta} \\ &= \text{RHS} \end{aligned}$$