

TRIGONOMETRY

- 1** **a** Write down the identities for $\sin(A + B)$ and $\sin(A - B)$.
b Hence, express $2 \sin A \cos B$ in terms of $\sin(A + B)$ and $\sin(A - B)$.
c Use the identities for $\cos(A + B)$ and $\cos(A - B)$ to obtain similar expressions for $2 \cos A \cos B$ and $2 \sin A \sin B$.

- 2** Express each of the following as the sum or difference of trigonometric functions.

- | | |
|------------------------------------------|------------------------------------------|
| a $2 \sin 30^\circ \cos 10^\circ$ | b $2 \cos 36^\circ \cos 18^\circ$ |
| c $\cos 49^\circ \sin 25^\circ$ | d $2 \sin 3A \sin A$ |
| e $2 \cos 5A \sin 2A$ | f $4 \cos 3A \cos B$ |
| g $\sin A \cos 6B$ | h $2 \cos A \sin(A + 40^\circ)$ |

- 3** **a** Use the identity for $2 \sin A \cos B$ to prove that

$$\sin P + \sin Q \equiv 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}.$$

- b** Obtain similar identities for

- i** $\sin P - \sin Q$
ii $\cos P + \cos Q$
iii $\cos P - \cos Q$

- 4** Express each of the following as the product of trigonometric functions.

- | | |
|------------------------------------------|----------------------------------------------------|
| a $\cos 25^\circ + \cos 15^\circ$ | b $\sin 84^\circ - \sin 30^\circ$ |
| c $\sin 5A + \sin A$ | d $\cos A - \cos 2A$ |
| e $\cos 2A - \cos 4B$ | f $\sin(A + 30^\circ) + \sin(A + 60^\circ)$ |
| g $2 \cos A + 2 \cos 3A$ | h $\sin(A + 2B) - \sin(3A - B)$ |

- 5** Solve each equation for x in the interval $0 \leq x \leq \pi$.

Give your answers to 2 decimal places where appropriate.

- | | |
|------------------------------------------|------------------------------------------------------------------|
| a $\sin 3x - \sin x = 0$ | b $\cos x = \cos 4x$ |
| c $2 \sin x \sin 5x = \cos 4x$ | d $8 \cos(x + \frac{\pi}{3}) \sin(x + \frac{\pi}{6}) = 1$ |
| e $\sin x + \sin \frac{x}{2} = 0$ | f $\cos 3x + \cos x = \cos 2x$ |

- 6** Solve each equation for x in the interval $0 \leq x \leq 180^\circ$.

- | | |
|-------------------------------------------|------------------------------------------------|
| a $2 \cos 2x \cos 3x - \cos x = 0$ | b $\sin 3x - \sin 2x = 0$ |
| c $\sin 4x + \sin 2x = \sin 3x$ | d $\cos 2x = \cos(x - 60^\circ)$ |
| e $\cos 5x \sin x + \sin 4x = 0$ | f $\sin x + \sin 3x = \cos x + \cos 3x$ |

- 7** Prove each identity.

- a** $\sin x + \sin 2x + \sin 3x \equiv \sin 2x (2 \cos x + 1)$
b $\frac{\cos x - \cos 3x}{\cos x + \cos 3x} \equiv \tan x \tan 2x$