


**TRIGONOMETRY**
**Answers**

- 1**
- a**  $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$  (1)  
 $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$  (2)
- b** let  $B = -B$  in (1)  $\Rightarrow$   $\sin[A + (-B)] \equiv \sin A \cos(-B) + \cos A \sin(-B)$   
 $\sin(A - B) \equiv \sin A \cos B + \cos A (-\sin B)$   
 $\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$
- let  $B = -B$  in (2)  $\Rightarrow$   $\cos[A + (-B)] \equiv \cos A \cos(-B) - \sin A \sin(-B)$   
 $\cos(A - B) \equiv \cos A \cos B - \sin A (-\sin B)$   
 $\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$
- c** (1)  $\div$  (2)  $\Rightarrow$   $\frac{\sin(A+B)}{\cos(A+B)} \equiv \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$
- $$\tan(A+B) \equiv \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$$
- $$\tan(A+B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
- let  $B = -B \Rightarrow$   $\tan[A + (-B)] \equiv \frac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)}$
- $$\tan(A - B) \equiv \frac{\tan A + (-\tan B)}{1 - \tan A(-\tan B)}$$
- $$\tan(A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$$
- 
- 2**
- a**  $= \sin(10 + 30)^\circ$   
 $= \sin 40^\circ$
- b**  $= \sin(67 - 18)^\circ$   
 $= \sin 49^\circ$
- c**  $= \sin(62 + 74)^\circ$   
 $= \sin 136^\circ$   
 $= \sin(180 - 136)^\circ$   
 $= \sin 44^\circ$
- d**  $= \cos(14 + 39)^\circ$   
 $= \cos 53^\circ$   
 $= \sin(90 - 53)^\circ$   
 $= \sin 37^\circ$
- 
- 3**
- a**  $= \cos(A + 2A)$   
 $= \cos 3A$
- b**  $= \sin(4A - B)$
- c**  $= \tan(2A + 5A)$   
 $= \tan 7A$
- d**  $= \cos(A - 3A)$   
 $= \cos(-2A)$   
 $= \cos 2A$

$$\begin{aligned}
 4 \quad \mathbf{a} &= \sin(45 - 30)^\circ \\
 &= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\
 &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \\
 &= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}(\sqrt{6} - \sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= \frac{1}{\sin 15^\circ} \\
 &= \frac{2\sqrt{2}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
 &= \frac{2\sqrt{2}(\sqrt{3}+1)}{3-1} \\
 &= \sqrt{6} + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} &= \cos(45 - 30)^\circ \\
 &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\
 &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\
 &= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 &= \frac{1}{4}(\sqrt{6} + \sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{g} &= \tan(30 + 45)^\circ \\
 &= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} \\
 &= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \times 1} \\
 &= \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\
 &= \frac{1 + 2\sqrt{3} + 3}{3 - 1} = 2 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \mathbf{a} &= \cos(x - 30^\circ) \\
 &\therefore \text{max.} = 1 \text{ when } x = 30^\circ \\
 \mathbf{c} &= \sin(x - 67^\circ) \\
 &\therefore \text{max.} = 1 \text{ when } x = 157^\circ
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \mathbf{a} &= \sin\left(x - \frac{\pi}{3}\right) \\
 &\therefore \text{min.} = -1 \text{ when } x = \frac{11\pi}{6} \\
 \mathbf{c} &= \cos(4x - x) \\
 &= \cos 3x \\
 &\therefore \text{min.} = -1 \text{ when } x = \frac{\pi}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= \sin 15^\circ \\
 &= \frac{1}{4}(\sqrt{6} - \sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= \sin(90 - 75)^\circ \\
 &= \sin 15^\circ \\
 &= \frac{1}{4}(\sqrt{6} - \sqrt{2})
 \end{aligned}$$

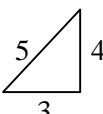
$$\begin{aligned}
 \mathbf{f} &= \frac{1}{\cos 195^\circ} \\
 &= \frac{1}{-\cos 15^\circ} \\
 &= -\frac{2\sqrt{2}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\
 &= -\frac{2\sqrt{2}(\sqrt{3}-1)}{3-1} \\
 &= \sqrt{2} - \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{h} &= \frac{1}{\sin 105^\circ} \\
 &= \frac{1}{\sin 75^\circ} \\
 &= \frac{1}{\cos 15^\circ} \\
 &= \sqrt{6} - \sqrt{2}
 \end{aligned}$$

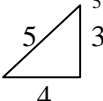
$$\begin{aligned}
 \mathbf{b} &= 3 \sin(x + 45^\circ) \\
 &\therefore \text{max.} = 3 \text{ when } x = 45^\circ \\
 \mathbf{d} &= -4(\cos x \cos 108^\circ - \sin x \sin 108^\circ) \\
 &= -4 \cos(x + 108^\circ) \\
 &\therefore \text{max.} = 4 \text{ when } x = 72^\circ
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= 2 \cos\left(x + \frac{\pi}{6}\right) \\
 &\therefore \text{min.} = -2 \text{ when } x = \frac{5\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= 6 \sin(2x - 3x) \\
 &= 6 \sin(-x) \\
 &= -6 \sin x \\
 &\therefore \text{min.} = -6 \text{ when } x = \frac{\pi}{2}
 \end{aligned}$$

7 a   $\therefore \tan A = \pm \frac{4}{3}$   
 $0 < A < 90^\circ \Rightarrow \tan A = \frac{4}{3}$

c  $= \cos A \cos B - \sin A \sin B$   
 $= \frac{3}{5} \times \frac{2}{3} - \frac{4}{5} \times \frac{\sqrt{5}}{3}$   
 $= \frac{2}{15}(3 - 2\sqrt{5})$

8 a   $\therefore \cos C = \pm \frac{4}{5}$   
 $0 < C < 90^\circ \Rightarrow \cos C = \frac{4}{5}$

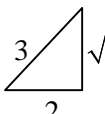
c  $= \sin C \cos D - \cos C \sin D$   
 $= \frac{3}{5} \times (-\frac{12}{13}) - \frac{4}{5} \times \frac{5}{13}$   
 $= -\frac{56}{65}$

9 a  $\sin(\theta + 15) = 0.4$   
 $\theta + 15 = 23.6, 180 - 23.6$   
 $= 23.6, 156.4$   
 $\theta = 8.6, 141.4$

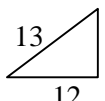
c  $\cos \theta \cos 60 + \sin \theta \sin 60 = \sin \theta$   
 $\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta = \sin \theta$   
 $(1 - \frac{\sqrt{3}}{2}) \sin \theta = \frac{1}{2} \cos \theta$   
 $\tan \theta = \frac{1}{2} \div (1 - \frac{\sqrt{3}}{2}) = 3.7321$   
 $\theta = 75, 180 + 75$   
 $\theta = 75, 255$

e  $\sin \theta \cos 30 + \cos \theta \sin 30$   
 $= \cos \theta \cos 45 + \sin \theta \sin 45$   
 $\frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta = \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta$   
 $(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}) \sin \theta = (\frac{1}{\sqrt{2}} - \frac{1}{2}) \cos \theta$   
 $\tan \theta = (\frac{1}{\sqrt{2}} - \frac{1}{2}) \div (\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}) = 1.3032$   
 $\theta = 52.2, 180 + 52.5$   
 $\theta = 52.5, 232.5$

10 LHS  $= \cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3} - (\cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3})$   
 $= -2 \sin x \sin \frac{\pi}{3}$   
 $= -\sqrt{3} \sin x \therefore k = -\sqrt{3}$

b   $\therefore \sin B = \pm \frac{\sqrt{5}}{3}$   
 $0 < B < 90^\circ \Rightarrow \sin B = \frac{\sqrt{5}}{3}$

d  $= \sin A \cos B + \cos A \sin B$   
 $= \frac{4}{5} \times \frac{2}{3} + \frac{3}{5} \times \frac{\sqrt{5}}{3}$   
 $= \frac{1}{15}(8 + 3\sqrt{5})$

b   $\therefore \cos D = \pm \frac{12}{13}$   
 $90^\circ < D < 180^\circ \Rightarrow \cos D = -\frac{12}{13}$

d  $\cos(C - D) = \cos C \cos D + \sin C \sin D$   
 $= \frac{4}{5} \times (-\frac{12}{13}) + \frac{3}{5} \times \frac{5}{13}$   
 $= -\frac{33}{65}$   
 $\therefore \sec(C - D) = -\frac{65}{33}$

b  $\tan(2\theta - 60) = 1$   
 $2\theta - 60 = 45, 180 + 45, 360 + 45, 540 + 45$   
 $= 45, 225, 405, 585$   
 $2\theta = 105, 285, 465, 645$   
 $\theta = 52.5, 142.5, 232.5, 322.5$

d  $2 \sin \theta + \sin \theta \cos 45 + \cos \theta \sin 45 = 0$   
 $2 \sin \theta + \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = 0$   
 $(2 + \frac{1}{\sqrt{2}}) \sin \theta = -\frac{1}{\sqrt{2}} \cos \theta$   
 $\tan \theta = -\frac{1}{\sqrt{2}} \div (2 + \frac{1}{\sqrt{2}}) = -0.2612$   
 $\theta = 180 - 14.6, 360 - 14.6$   
 $\theta = 165.4, 345.4$

f  $3(\cos 2\theta \cos 60 - \sin 2\theta \sin 60)$   
 $- (\sin 2\theta \cos 30 - \cos 2\theta \sin 30) = 0$   
 $\frac{3}{2} \cos 2\theta - \frac{3\sqrt{3}}{2} \sin 2\theta - \frac{\sqrt{3}}{2} \sin 2\theta + \frac{1}{2} \cos 2\theta = 0$   
 $2\sqrt{3} \sin 2\theta = 2 \cos 2\theta$   
 $\tan 2\theta = \frac{1}{\sqrt{3}}$   
 $2\theta = 30, 180 + 30, 360 + 30, 540 + 30$   
 $= 30, 210, 390, 570$   
 $\theta = 15, 105, 195, 285$

- 11 a**  $\text{LHS} = \cos x - (\cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3})$   
 $= \cos x - \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$   
 $= \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$   
 $= \cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}$   
 $= \cos (x + \frac{\pi}{3}) = \text{RHS}$
- b**  $\text{LHS} = \sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6} + \cos x$   
 $= \frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x + \cos x$   
 $= \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x$   
 $= \sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6}$   
 $= \sin (x + \frac{\pi}{6}) = \text{RHS}$
- 12 a**  $\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$   
 let  $B = A \Rightarrow \sin (A + A) \equiv \sin A \cos A + \cos A \sin A$   
 $\sin 2A \equiv 2 \sin A \cos A$
- b**  $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$   
 let  $B = A \Rightarrow \cos (A + A) \equiv \cos A \cos A - \sin A \sin A$   
 $\cos 2A \equiv \cos^2 A - \sin^2 A$
- c i**  $\cos 2A \equiv \cos^2 A - (1 - \cos^2 A)$   
 $\cos 2A \equiv 2 \cos^2 A - 1$
- ii**  $\cos 2A \equiv 1 - \sin^2 A - \sin^2 A$   
 $\cos 2A \equiv 1 - 2 \sin^2 A$
- d**  $\tan (A + B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$   
 let  $B = A \Rightarrow \tan (A + A) \equiv \frac{\tan A + \tan A}{1 - \tan A \tan A}$   
 $\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$
- 13 a**  $2 \cos^2 x - 1 + \cos x = 0$   
 $(2 \cos x - 1)(\cos x + 1) = 0$   
 $\cos x = -1$  or  $\frac{1}{2}$   
 $x = 180$  or  $60, 360 - 60$   
 $x = 60^\circ, 180^\circ, 300^\circ$
- b**  $2 \sin x \cos x + \cos x = 0$   
 $\cos x(2 \sin x + 1) = 0$   
 $\cos x = 0$  or  $\sin x = -\frac{1}{2}$   
 $x = 90, 360 - 90$  or  $180 + 30, 360 - 30$   
 $x = 90^\circ, 210^\circ, 270^\circ, 330^\circ$
- c**  $2(1 - 2 \sin^2 x) = 7 \sin x$   
 $4 \sin^2 x + 7 \sin x - 2 = 0$   
 $(4 \sin x - 1)(\sin x + 2) = 0$   
 $\sin x = \frac{1}{4}$  or  $-2$  [no solutions]  
 $x = 14.5, 180 - 14.5$   
 $x = 14.5^\circ, 165.5^\circ$
- d**  $11 \cos x = 4 + 3(2 \cos^2 x - 1)$   
 $6 \cos^2 x - 11 \cos x + 1 = 0$   
 $\cos x = \frac{11 \pm \sqrt{121 - 24}}{12} = \frac{11 \pm \sqrt{97}}{12}$   
 $\cos x = 0.09593$  or  $1.7374$  [no solutions]  
 $x = 84.5, 360 - 84.5$   
 $x = 84.5^\circ, 275.5^\circ$

$$\begin{aligned} \mathbf{e} \quad & \frac{2 \tan x}{1 - \tan^2 x} - \tan x = 0 \\ & 2 \tan x = \tan x(1 - \tan^2 x) \\ & \tan^3 x + \tan x = 0 \\ & \tan x(\tan^2 x + 1) = 0 \\ & \tan x = 0 \text{ or } \tan^2 x = -1 \text{ [no solutions]} \\ & x = 0, 180^\circ, 360^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{g} \quad & 10 \sin 2x \cos 2x = 2 \sin 2x \\ & 2 \sin 2x(5 \cos 2x - 1) = 0 \\ & \sin 2x = 0 \text{ or } \cos 2x = \frac{1}{5} \\ & 2x = 0, 180, 360, 540, 720 \\ & \quad \text{or } 78.463, 360 - 78.463, \\ & \quad \quad 360 + 78.463, 720 - 78.463 \\ & = 0, 78.463, 180, 281.537, 360 \\ & \quad 438.463, 540, 641.537, 720 \\ & x = 0, 39.2^\circ, 90^\circ, 140.8^\circ, 180^\circ, \\ & \quad 219.2^\circ, 270^\circ, 320.8^\circ, 360^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{14} \quad \mathbf{a} \quad \text{LHS} &= \cos^2 x + 2 \sin x \cos x + \sin^2 x \\ &= \cos^2 x + \sin^2 x + \sin 2x \\ &= 1 + \sin 2x \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad \text{LHS} &= \frac{2 \sin x \cos x}{\cos x(2 \cos x - \sec x)} \\ &= \frac{2 \sin x \cos x}{2 \cos^2 x - 1} \\ &= \frac{\sin 2x}{\cos 2x} \\ &= \tan 2x \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad \text{LHS} &= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x} \\ &= \frac{1 - \cos 2x}{\sin 2x} \\ &= \frac{1 - (1 - 2 \sin^2 x)}{\sin 2x} \\ &= \frac{2 \sin^2 x}{2 \sin x \cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad & \frac{1}{\cos x} = 4 \sin x \\ & 1 = 4 \sin x \cos x \\ & 1 = 2 \sin 2x \\ & \sin 2x = \frac{1}{2} \\ & 2x = 30, 180 - 30, 360 + 30, 540 - 30 \\ & \quad = 30, 150, 390, 510 \\ & x = 15^\circ, 75^\circ, 195^\circ, 255^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{h} \quad & 2(1 - \cos^2 x) - (2 \cos^2 x - 1) - \cos x = 0 \\ & 4 \cos^2 x + \cos x - 3 = 0 \\ & (4 \cos x - 3)(\cos x + 1) = 0 \\ & \cos x = -1 \text{ or } \frac{3}{4} \\ & x = 180 \text{ or } 41.4, 360 - 41.4 \\ & x = 41.4^\circ, 180, 318.6^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \text{LHS} &= \tan x(1 + 2 \cos^2 x - 1) \\ &= \frac{\sin x}{\cos x} \times 2 \cos^2 x \\ &= 2 \sin x \cos x \\ &= \sin 2x \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad \text{LHS} &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \\ &= \frac{1}{\frac{1}{2} \sin 2x} \\ &= 2 \operatorname{cosec} 2x \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad \text{LHS} &= \cos x \operatorname{cosec} x - 1 + 1 - \sin x \sec x \\ &= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \\ &= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \\ &= \frac{\cos 2x}{\frac{1}{2} \sin 2x} \\ &= 2 \cot 2x \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned}
 \mathbf{g} \quad \text{LHS} &= \frac{\sin x(1 - \sin 2x)}{\sin x(\operatorname{cosec} x - 2\cos x)} \\
 &= \frac{\sin x(1 - \sin 2x)}{1 - 2\sin x \cos x} \\
 &= \frac{\sin x(1 - \sin 2x)}{1 - \sin 2x} \\
 &= \sin x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{h} \quad \text{LHS} &= \cos(2x + x) \\
 &= \cos 2x \cos x - \sin 2x \sin x \\
 &= \cos x(2\cos^2 x - 1) - 2\sin^2 x \cos x \\
 &= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) \\
 &= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x \\
 &= 4\cos^3 x - 3\cos x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{15} \quad \mathbf{a} \quad \cos 2A &\equiv 2\cos^2 A - 1 \\
 \text{let } A &= \frac{x}{2} \\
 \cos x &\equiv 2\cos^2 \frac{x}{2} - 1
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \cos 2A &\equiv 1 - 2\sin^2 A \\
 \text{let } A &= \frac{x}{2} \\
 \cos x &\equiv 1 - 2\sin^2 \frac{x}{2} \\
 \sin^2 \frac{x}{2} &\equiv \frac{1}{2}(1 - \cos x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{16} \quad \mathbf{a} \quad \sin^2 \frac{A}{2} &= \frac{1}{2}\left(1 - \frac{7}{9}\right) = \frac{1}{9} \\
 \sin \frac{A}{2} &= \pm \frac{1}{3} \\
 0 < \frac{A}{2} < 45^\circ &\therefore \sin \frac{A}{2} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad -\frac{3}{8} &= 2\cos^2 \frac{B}{2} - 1 \\
 \cos^2 \frac{B}{2} &= \frac{1}{2}\left(-\frac{3}{8} + 1\right) = \frac{5}{16} \\
 \cos \frac{B}{2} &= \pm \frac{1}{4}\sqrt{5} \\
 45^\circ < \frac{B}{2} < 90^\circ &\therefore \cos \frac{B}{2} = \frac{1}{4}\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{17} \quad \mathbf{a} \quad \text{LHS} &= \frac{2}{1 + (2\cos^2 \frac{x}{2} - 1)} \\
 &= \frac{2}{2\cos^2 \frac{x}{2}} \\
 &= \sec^2 \frac{x}{2} \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \text{LHS} &= \frac{1 + (2\cos^2 \frac{x}{2} - 1)}{1 - (1 - 2\sin^2 \frac{x}{2})} \\
 &= \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} \\
 &= \cot^2 \frac{x}{2} \\
 &= \text{RHS}
 \end{aligned}$$