

TRIGONOMETRY

- 1** **a** Write down the identities for $\sin(A + B)$ and $\cos(A + B)$.
b Use these identities to obtain similar identities for $\sin(A - B)$ and $\cos(A - B)$.
c Use the above identities to obtain similar identities for $\tan(A + B)$ and $\tan(A - B)$.
- 2** Express each of the following in the form $\sin \alpha$, where α is acute.
- a** $\sin 10^\circ \cos 30^\circ + \cos 10^\circ \sin 30^\circ$ **b** $\sin 67^\circ \cos 18^\circ - \cos 67^\circ \sin 18^\circ$
c $\sin 62^\circ \cos 74^\circ + \cos 62^\circ \sin 74^\circ$ **d** $\cos 14^\circ \cos 39^\circ - \sin 14^\circ \sin 39^\circ$
- 3** Express as a single trigonometric ratio
- a** $\cos A \cos 2A - \sin A \sin 2A$ **b** $\sin 4A \cos B - \cos 4A \sin B$
c $\frac{\tan 2A + \tan 5A}{1 - \tan 2A \tan 5A}$ **d** $\cos A \cos 3A + \sin A \sin 3A$
- 4** Find in exact form, with a rational denominator, the value of
- a** $\sin 15^\circ$ **b** $\sin 165^\circ$ **c** $\operatorname{cosec} 15^\circ$ **d** $\cos 75^\circ$
e $\cos 15^\circ$ **f** $\sec 195^\circ$ **g** $\tan 75^\circ$ **h** $\operatorname{cosec} 105^\circ$
- 5** Find the maximum value that each expression can take and the smallest positive value of x , in degrees, for which this maximum occurs.
- a** $\cos x \cos 30^\circ + \sin x \sin 30^\circ$ **b** $3 \sin x \cos 45^\circ + 3 \cos x \sin 45^\circ$
c $\sin x \cos 67^\circ - \cos x \sin 67^\circ$ **d** $4 \sin x \sin 108^\circ - 4 \cos x \cos 108^\circ$
- 6** Find the minimum value that each expression can take and the smallest positive value of x , in radians in terms of π , for which this minimum occurs.
- a** $\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3}$ **b** $2 \cos x \cos \frac{\pi}{6} - 2 \sin x \sin \frac{\pi}{6}$
c $\cos 4x \cos x + \sin 4x \sin x$ **d** $6 \sin 2x \cos 3x - 6 \sin 3x \cos 2x$
- 7** Given that $\sin A = \frac{4}{5}$, $0 < A < 90^\circ$ and that $\cos B = \frac{2}{3}$, $0 < B < 90^\circ$, find without using a calculator the value of
- a** $\tan A$ **b** $\sin B$ **c** $\cos(A + B)$ **d** $\sin(A + B)$
- 8** Given that $\operatorname{cosec} C = \frac{5}{3}$, $0 < C < 90^\circ$ and that $\sin D = \frac{5}{13}$, $90^\circ < D < 180^\circ$, find without using a calculator the value of
- a** $\cos C$ **b** $\cos D$ **c** $\sin(C - D)$ **d** $\sec(C - D)$
- 9** Solve each equation for θ in the interval $0 \leq \theta \leq 360$.
Give your answers to 1 decimal place where appropriate.
- a** $\sin \theta^\circ \cos 15^\circ + \cos \theta^\circ \sin 15^\circ = 0.4$ **b** $\frac{\tan 2\theta^\circ - \tan 60^\circ}{1 + \tan 2\theta^\circ \tan 60^\circ} = 1$
c $\cos(\theta - 60)^\circ = \sin \theta^\circ$ **d** $2 \sin \theta^\circ + \sin(\theta + 45)^\circ = 0$
e $\sin(\theta + 30)^\circ = \cos(\theta - 45)^\circ$ **f** $3 \cos(2\theta + 60)^\circ - \sin(2\theta - 30)^\circ = 0$

- 10 Find the value of k such that for all real values of x

$$\cos\left(x + \frac{\pi}{3}\right) - \cos\left(x - \frac{\pi}{3}\right) \equiv k \sin x.$$

- 11 Prove each identity.

a $\cos x - \cos\left(x - \frac{\pi}{3}\right) \equiv \cos\left(x + \frac{\pi}{3}\right)$

b $\sin\left(x - \frac{\pi}{6}\right) + \cos x \equiv \sin\left(x + \frac{\pi}{6}\right)$

- 12 a Use the identity for $\sin(A + B)$ to express $\sin 2A$ in terms of $\sin A$ and $\cos A$.
 b Use the identity for $\cos(A + B)$ to express $\cos 2A$ in terms of $\sin A$ and $\cos A$.
 c Hence, express $\cos 2A$ in terms of
 i $\cos A$ ii $\sin A$
 d Use the identity for $\tan(A + B)$ to express $\tan 2A$ in terms of $\tan A$.

- 13 Solve each equation for x in the interval $0 \leq x \leq 360^\circ$.

Give your answers to 1 decimal place where appropriate.

a $\cos 2x + \cos x = 0$

b $\sin 2x + \cos x = 0$

c $2 \cos 2x = 7 \sin x$

d $11 \cos x = 4 + 3 \cos 2x$

e $\tan 2x - \tan x = 0$

f $\sec x - 4 \sin x = 0$

g $5 \sin 4x = 2 \sin 2x$

h $2 \sin^2 x - \cos 2x - \cos x = 0$

- 14 Prove each identity.

a $(\cos x + \sin x)^2 \equiv 1 + \sin 2x$

b $\tan x (1 + \cos 2x) \equiv \sin 2x$

c $\frac{2 \sin x}{2 \cos x - \sec x} \equiv \tan 2x$

d $\tan x + \cot x \equiv 2 \operatorname{cosec} 2x$

e $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$

f $(\cos x + \sin x)(\operatorname{cosec} x - \sec x) \equiv 2 \cot 2x$

g $\frac{1 - \sin 2x}{\operatorname{cosec} x - 2 \cos x} \equiv \sin x$

h $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$

- 15 Use the double angle identities to prove that

a $\cos x \equiv 2 \cos^2 \frac{x}{2} - 1$

b $\sin^2 \frac{x}{2} \equiv \frac{1}{2}(1 - \cos x)$

- 16 a Given that $\cos A = \frac{7}{9}$, $0 < A < 90^\circ$, find the exact value of $\sin \frac{A}{2}$ without using a calculator.
 b Given that $\cos B = -\frac{3}{8}$, $90^\circ < B < 180^\circ$, find the value of $\cos \frac{B}{2}$, giving your answer in the form $k\sqrt{5}$.

- 17 Prove each identity.

a $\frac{2}{1 + \cos x} \equiv \sec^2 \frac{x}{2}$

b $\frac{1 + \cos x}{1 - \cos x} \equiv \cot^2 \frac{x}{2}$