TRIGONOMETRY — Answers

1 **a** $\sin^2 x + \cos^2 x \equiv 1$

$\Rightarrow \quad \frac{\sin^2 x}{\cos^2 x} + 1 \equiv \frac{1}{\cos^2 x}$

$\Rightarrow \quad \tan^2 x + 1 \equiv \sec^2 x$

b $\sin^2 x + \cos^2 x \equiv 1$

$\Rightarrow \quad 1 + \frac{\cos^2 x}{\sin^2 x} \equiv \frac{1}{\sin^2 x}$

$\Rightarrow \quad 1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

2 **a** $\tan^2 A = \frac{1}{9}$

$\sec^2 A = 1 + \frac{1}{9} = \frac{10}{9}$

b $\operatorname{cosec}^2 B = 1 + 2\sqrt{3} + 3$

$= 4 + 2\sqrt{3}$

$\cot^2 B = (4 + 2\sqrt{3}) - 1$

$= 3 + 2\sqrt{3}$

c $\sec^2 C = \frac{9}{4}$

$\tan^2 C = \frac{9}{4} - 1 = \frac{5}{4}$

$\tan C = \pm\sqrt{\frac{5}{4}} = \pm\frac{1}{2}\sqrt{5}$

3 **a** $3(1 + \tan^2 \theta) = 4\tan^2 \theta$

$\tan^2 \theta = 3$

$\tan \theta = \pm\sqrt{3}$

$\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

b $\sec^2 \theta - 1 - 2\sec \theta + 1 = 0$

$\sec^2 \theta - 2\sec \theta = 0$

$\sec \theta(\sec \theta - 2) = 0$

$\sec \theta = 2$ or 0 [no solutions]

$\cos \theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

c $\operatorname{cosec}^2 \theta - 1 - 3\operatorname{cosec} \theta + 3 = 0$

$\operatorname{cosec}^2 \theta - 3\operatorname{cosec} \theta + 2 = 0$

$(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta - 2) = 0$

$\operatorname{cosec} \theta = 1$ or 2

$\sin \theta = \frac{1}{2}$ or 1

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

d $1 + \cot^2 \theta + \cot^2 \theta = 3$

$\cot^2 \theta = 1$

$\cot \theta = \pm 1$

$\tan \theta = \pm 1$

$\theta = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

e $1 + \tan^2 \theta + 2\tan \theta = 0$

$(\tan \theta + 1)^2 = 0$

$\tan \theta = -1$

$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

f $1 + \cot^2 \theta - \sqrt{3}\cot \theta - 1 = 0$

$\cot^2 \theta - \sqrt{3}\cot \theta = 0$

$\cot \theta(\cot \theta - \sqrt{3}) = 0$

$\cot \theta = 0$ or $\sqrt{3}$

$\cos \theta = 0$ or $\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$ or $\frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$

4 **a** $\sec^2 x - 1 - 2\sec x - 2 = 0$
$\sec^2 x - 2\sec x - 3 = 0$
$(\sec x + 1)(\sec x - 3) = 0$
$\sec x = -1$ or 3
$\cos x = -1$ or $\frac{1}{3}$
$x = 180, -180$ or $70.5, -70.5$
$x = -180°, -70.5°, 70.5°, 180°$

b $2(1 + \cot^2 x) + 2 = 9\cot x$
$2\cot^2 x - 9\cot x + 4 = 0$
$(2\cot x - 1)(\cot x - 4) = 0$
$\cot x = \frac{1}{2}$ or 4
$\tan x = \frac{1}{4}$ or 2
$x = 14.0, 14.0 - 180$ or $63.4, 63.4 - 180$
$x = -166.0°, -116.6°, 14.0°, 63.4°$

c $\operatorname{cosec}^2 x + 5\operatorname{cosec} x + 2(\operatorname{cosec}^2 x - 1) = 0$
$3\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 2 = 0$
$(3\operatorname{cosec} x - 1)(\operatorname{cosec} x + 2) = 0$
$\operatorname{cosec} x = -2$ or $\frac{1}{3}$ [no solutions]
$\sin x = -\frac{1}{2}$
$x = -30, 30 - 180$
$x = -150°, -30°$

d $3\tan^2 x - 3\tan x + 1 + \tan^2 x = 2$
$4\tan^2 x - 3\tan x - 1 = 0$
$(4\tan x + 1)(\tan x - 1) = 0$
$\tan x = -\frac{1}{4}$ or 1
$x = 180 - 14.0, -14.0$ or $45, 45 - 180$
$x = -135°, -14.0°, 45°, 166.0°$

e $\sec^2 x - 1 + 4\sec x - 2 = 0$
$\sec^2 x + 4\sec x - 3 = 0$
$\sec x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$
$\cos x = \frac{1}{-2 \pm \sqrt{7}}$
$\cos x = -0.2153$ or 1.5486 [no solutions]
$x = 180 - 77.6, 77.6 - 180$
$x = -102.4°, 102.4°$

f $2\cot^2 x + 3(1 + \cot^2 x) = 4\cot x + 3$
$5\cot^2 x - 4\cot x = 0$
$\cot x(5\cot x - 4) = 0$
$\cot x = 0$ or $\frac{4}{5}$
$\cos x = 0$ or $\tan x = \frac{5}{4}$
$x = 90, -90$ or $51.3, 51.3 - 180$
$x = -128.7°, -90°, 51.3°, 90°$

5 **a** $\operatorname{cosec}^2 2x - 1 + \operatorname{cosec} 2x - 1 = 0$
$\operatorname{cosec}^2 2x + \operatorname{cosec} 2x - 2 = 0$
$(\operatorname{cosec} 2x + 2)(\operatorname{cosec} 2x - 1) = 0$
$\operatorname{cosec} 2x = -2$ or 1
$\sin 2x = -\frac{1}{2}$ or 1
$2x = 180 + 30, 360 - 30, 540 + 30,$
$720 - 30$ or $90, 360 + 90$
$= 90, 210, 330, 450, 570, 690$
$x = 45°, 105°, 165°, 225°, 285°, 345°$

b $8(1 - \cos^2 x) + \sec x = 8$
$8\cos^2 x = \sec x$
$\cos^3 x = \frac{1}{8}$
$\cos x = \frac{1}{2}$
$x = 60, 360 - 60$
$x = 60°, 300°$

c $\frac{3}{\sin^2 x} - 4\sin^2 x = 1$
$4\sin^4 x + \sin^2 x - 3 = 0$
$(4\sin^2 x - 3)(\sin^2 x + 1) = 0$
$\sin^2 x = \frac{3}{4}$ or -1 [no solutions]
$\sin x = \pm\frac{\sqrt{3}}{2}$
$x = 60, 180 - 60$ or $180 + 60, 360 - 60$
$x = 60°, 120°, 240°, 300°$

d $9(1 + \tan^2 x) - 8 = 1 + \cot^2 x$
$9\tan^2 x = \cot^2 x$
$\tan^4 x = \frac{1}{9}$
$\tan^2 x = \frac{1}{3}$ or $-\frac{1}{3}$ [no solutions]
$\tan x = \pm\frac{1}{\sqrt{3}}$
$x = 30, 180 + 30$ or $180 - 30, 360 - 30$
$x = 30°, 150°, 210°, 330°$

6 **a** LHS = $1 + \cot^2 x - (1 + \tan^2 x)$
$= \cot^2 x - \tan^2 x$
= RHS

b LHS = $\cot^2 x - 2\cot x + 1$
$= \operatorname{cosec}^2 x - 2\cot x$
= RHS

c LHS = $\cos^2 x - 4 + 4\sec^2 x$
$= \cos^2 x - 4 + 4(1 + \tan^2 x)$
$= \cos^2 x + 4\tan^2 x$
= RHS

d LHS = $1 + \tan^2 x - (1 - \cos^2 x)$
$= \tan^2 x + \cos^2 x$
= RHS

e LHS = $\tan^2 x + 2 + \cot^2 x$
$= \sec^2 - 1 + 2 + \operatorname{cosec}^2 x - 1$
$= \sec^2 x + \operatorname{cosec}^2 x$
= RHS

f LHS = $\sin^2 x - 2\sin x \sec x + \sec^2 x$
$= \sin^2 x - 2\tan x + 1 + \tan^2 x$
$= \sin^2 x + (\tan x - 1)^2$
= RHS

g LHS = $\dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$
$= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$
$= \dfrac{1}{\cos^2 x \sin^2 x}$
$= \dfrac{1}{\cos^2 x} \times \dfrac{1}{\sin^2 x}$
$= \sec^2 x \operatorname{cosec}^2 x$
= RHS

h LHS = $\sec^2 x (1 + \tan^2 x) + \tan^2 x (\sec^2 x - 1)$
$= \sec^2 x + \sec^2 x \tan^2 x + \sec^2 x \tan^2 x - \tan^2 x$
$= 1 + \tan^2 x + 2\sec^2 x \tan^2 x - \tan^2 x$
$= 2\sec^2 x \tan^2 x + 1$
= RHS

7 $4\sec^2 x - \sec x + 2\tan^2 x = 0 \Rightarrow 4\sec^2 x - \sec x + 2(\sec^2 x - 1) = 0$
$6\sec^2 x - \sec x - 2 = 0$
$(3\sec x - 2)(2\sec x + 1) = 0$
$\sec x = -\frac{1}{2}, \frac{2}{3}$
for real values of x, $|\sec x| > 1$ $\therefore$ no real solutions

8 **a** LHS = $\dfrac{1}{\sin x} \times \dfrac{1}{\cos x} - \dfrac{\cos x}{\sin x}$
$= \dfrac{1 - \cos^2 x}{\sin x \cos x}$
$= \dfrac{\sin^2 x}{\sin x \cos x}$
$= \dfrac{\sin x}{\cos x}$
$= \tan x$
= RHS

b $\operatorname{cosec} x \sec x - \cot x = 3$
$\tan x = 3$
$x = 71.6, 180 + 71.6$
$x = 71.6°, 251.6°$