

TRIGONOMETRY

Answers

1 **a** 1.09

b -11.47

c 0.33

d 1.89

a $= 1 \div \sin 30^\circ$
 $= 1 \div \frac{1}{2}$
 $= 2$

b $= 1 \div \tan 45^\circ$
 $= 1 \div 1$
 $= 1$

c $= 1 \div \cos 150^\circ$
 $= 1 \div (-\cos 30^\circ)$
 $= 1 \div (-\frac{\sqrt{3}}{2})$
 $= -\frac{2}{\sqrt{3}}$

d $= 1 \div \sin 300^\circ$
 $= 1 \div (-\sin 60^\circ)$
 $= 1 \div (-\frac{\sqrt{3}}{2})$
 $= -\frac{2}{\sqrt{3}}$

e $= \cos 90^\circ \div \sin 90^\circ$
 $= 0 \div 1$
 $= 0$

f $= 1 \div \cos 225^\circ$
 $= 1 \div (-\cos 45^\circ)$
 $= 1 \div (-\frac{1}{\sqrt{2}})$
 $= -\sqrt{2}$

g $= 1 \div \sin 270^\circ$
 $= 1 \div (-\sin 90^\circ)$
 $= 1 \div (-1)$
 $= -1$

h $= 1 \div \tan 330^\circ$
 $= 1 \div (-\tan 30^\circ)$
 $= 1 \div (-\frac{1}{\sqrt{3}})$
 $= -\sqrt{3}$

i $= 1 \div \cos 660^\circ$
 $= 1 \div \cos 60^\circ$
 $= 1 \div \frac{1}{2}$
 $= 2$

j $= 1 \div \sin (-45^\circ)$
 $= 1 \div (-\sin 45^\circ)$
 $= 1 \div (-\frac{1}{\sqrt{2}})$
 $= -\sqrt{2}$

k $= 1 \div \tan (-240^\circ)$
 $= 1 \div (-\tan 60^\circ)$
 $= 1 \div (-\sqrt{3})$
 $= -\frac{1}{\sqrt{3}}$

l $= 1 \div \cos (-315^\circ)$
 $= 1 \div \cos 45^\circ$
 $= 1 \div \frac{1}{\sqrt{2}}$
 $= \sqrt{2}$

3 **a** 1.60

b 1.01

c -2.09

d 2.54

a $= 1 \div \cos 0$
 $= 1 \div 1$
 $= 1$

b $= 1 \div \sin \frac{\pi}{4}$
 $= 1 \div \frac{1}{\sqrt{2}}$
 $= \sqrt{2}$

c $= 1 \div \tan \frac{3\pi}{4}$
 $= 1 \div (-\tan \frac{\pi}{4})$
 $= 1 \div (-1)$
 $= -1$

d $= 1 \div \cos \frac{4\pi}{3}$
 $= 1 \div (-\cos \frac{\pi}{3})$
 $= 1 \div (-\frac{1}{2})$
 $= -2$

e $= 1 \div \sin \frac{2\pi}{3}$
 $= 1 \div \sin \frac{\pi}{3}$
 $= 1 \div \frac{\sqrt{3}}{2}$
 $= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
 $= \frac{2}{3}\sqrt{3}$

f $= \cos \frac{7\pi}{2} \div \sin \frac{7\pi}{2}$
 $= \cos \frac{\pi}{2} \div (-\sin \frac{\pi}{2})$
 $= 0 \div (-1)$
 $= 0$

g $= 1 \div \cos \frac{5\pi}{4}$
 $= 1 \div (-\cos \frac{\pi}{4})$
 $= 1 \div (-\frac{1}{\sqrt{2}})$
 $= -\sqrt{2}$

h $= 1 \div \sin (-\frac{5\pi}{6})$
 $= 1 \div (-\sin \frac{\pi}{6})$
 $= 1 \div (-\frac{1}{2})$
 $= -2$

i $= 1 \div \tan \frac{11\pi}{6}$
 $= 1 \div (-\tan \frac{\pi}{6})$
 $= 1 \div (-\frac{1}{\sqrt{3}})$
 $= -\sqrt{3}$

j $= 1 \div \cos (-4\pi)$
 $= 1 \div \cos 0$
 $= 1 \div 1$
 $= 1$

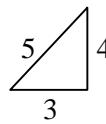
k $= 1 \div \sin \frac{13\pi}{4}$
 $= 1 \div (-\sin \frac{\pi}{4})$
 $= 1 \div (-\frac{1}{\sqrt{2}})$
 $= -\sqrt{2}$

l $= 1 \div \tan (-\frac{7\pi}{3})$
 $= 1 \div (-\tan \frac{\pi}{3})$
 $= 1 \div (-\sqrt{3})$
 $= -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
 $= -\frac{1}{3}\sqrt{3}$

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$$\therefore \cos x = \pm \frac{3}{5}, \tan x = \pm \frac{4}{3}$$

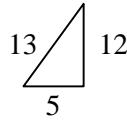
$$0 < x < 90^\circ \Rightarrow \cos x = \frac{3}{5}, \tan x = \frac{4}{3}$$

a $= \frac{3}{5}$

b $= \frac{4}{3}$

c $= 1 \div \frac{4}{5} = \frac{5}{4}$

d $= 1 \div \frac{3}{5} = \frac{5}{3}$

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$$\therefore \sin x = \pm \frac{12}{13}, \tan x = \pm \frac{12}{5}$$

$$90^\circ < x < 180^\circ \Rightarrow \sin x = -\frac{12}{13}, \tan x = -\frac{12}{5}$$

a $= \frac{12}{13}$

b $= 1 \div (-\frac{5}{13}) = -\frac{13}{5}$

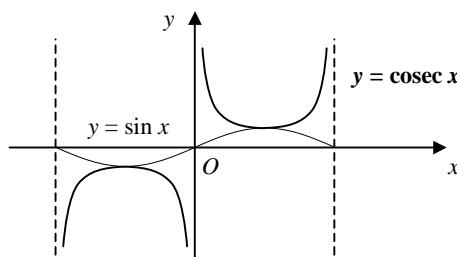
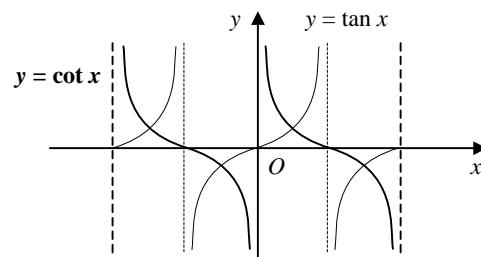
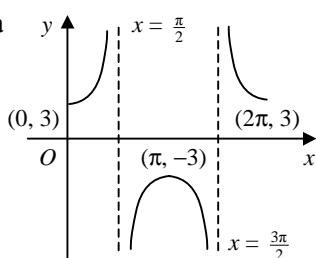
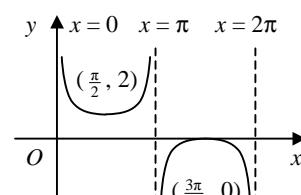
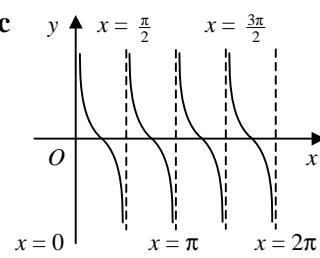
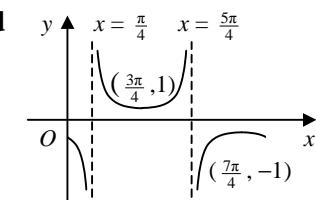
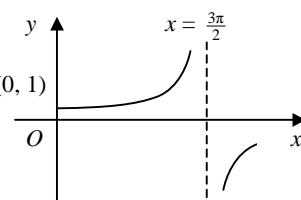
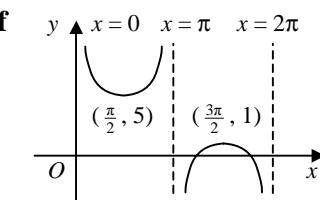
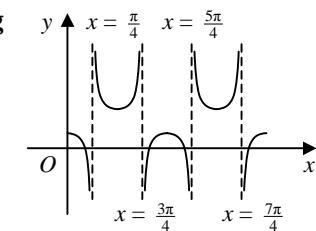
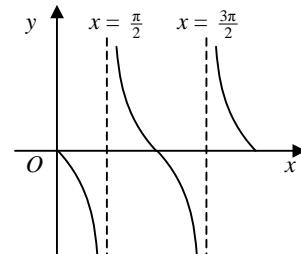
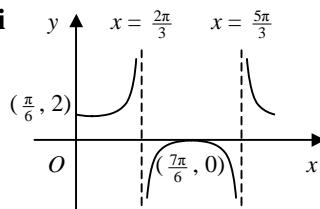
c $= 1 \div \frac{12}{13} = \frac{13}{12}$

d $= 1 \div -\frac{12}{5} = -\frac{5}{12}$

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a $(0, 1), (180, -1), (360, 1), (540, -1), (720, 1)$

b $x = 90, x = 270, x = 450, x = 630$

8**a****b****9****a****b****c****d****e****f****g****h****i**

TP: $(0, 0), (\frac{\pi}{2}, 2), (\pi, 0),$

$(\frac{3\pi}{2}, 2), (2\pi, 0)$

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- 10** **a** $\tan x = 1$
 $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$
- b** $\cos x = \frac{1}{2}$
 $x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$
 $x = \frac{\pi}{3}, \frac{5\pi}{3}$
- c** $\sin x = \frac{1}{\sqrt{2}}$
 $x = \frac{\pi}{4}, \pi - \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{3\pi}{4}$
- d** $\cos x = 0$
 $x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$
 $x = \frac{\pi}{2}, \frac{3\pi}{2}$
- e** $\cos x = -1$
 $x = \pi$
- f** $\sin x = -\frac{1}{2}$
 $x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
 $x = \frac{7\pi}{6}, \frac{11\pi}{6}$
- g** $\tan x = -\frac{1}{\sqrt{3}}$
 $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
 $x = \frac{5\pi}{6}, \frac{11\pi}{6}$
- h** $\cos x = -\frac{1}{\sqrt{2}}$
 $x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$
 $x = \frac{3\pi}{4}, \frac{5\pi}{4}$
- 11** **a** $\cos \theta = 0.5556$
 $\theta = 56.3, 360 - 56.3$
 $\theta = 56.3^\circ, 303.7^\circ$
- b** $\sin \theta = 0.3891$
 $\theta = 22.9, 180 - 22.9$
 $\theta = 22.9^\circ, 157.1^\circ$
- c** $\tan \theta = 0.9434$
 $\theta = 43.3, 180 + 43.3$
 $\theta = 43.3^\circ, 223.3^\circ$
- d** $\cos \theta = -0.3802$
 $\theta = 180 - 67.7,$
 $180 + 67.7$
 $\theta = 112.3^\circ, 247.7^\circ$
- e** $\sin \theta = 0.3333$
 $\theta = 19.5, 180 - 19.5$
 $\theta = 19.5^\circ, 160.5^\circ$
- f** $\tan \theta = -1.0638$
 $\theta = 180 - 46.8,$
 $360 - 46.8$
 $\theta = 133.2^\circ, 313.2^\circ$
- g** $\cos \theta = 0.5297$
 $\theta = 58.0, 360 - 58.0$
 $\theta = 58.0^\circ, 302.0^\circ$
- h** $\sin \theta = -0.8333$
 $\theta = 180 + 56.4,$
 $360 - 56.4$
 $\theta = 236.4^\circ, 303.6^\circ$
- 12** **a** $\sin(x + 30) = 0.5$
 $x + 30 = 30, 180 - 30$
 $= 30, 150$
 $x = 0, 120$
- b** $\tan(x - 57) = 0.625$
 $x - 57 = 32.0, 32.0 - 180$
 $= -148.0, 32.0$
 $x = -91.0, 89.0$
- c** $\cos 2x = 0.4255$
 $2x = 64.816, 360 - 64.816,$
 $-64.816, 64.816 - 360$
 $= -295.184, -64.816,$
 $64.816, 295.184$
 $x = -147.6, -32.4,$
 $32.4, 147.6$
- d** $\cot x = 2.5$
 $\tan x = 0.4$
 $x = 21.8, 21.8 - 180$
 $x = -158.2, 21.8$
- e** $\sec(x - 60) = \frac{2}{\sqrt{3}}$
 $\cos(x - 60) = \frac{\sqrt{3}}{2}$
 $x - 60 = 30, -30$
 $x = 30, 90$
- f** $\operatorname{cosec} \frac{1}{2}x = 3.5$
 $\sin \frac{1}{2}x = 0.2857$
 $\frac{1}{2}x = 16.602$
 $x = 33.2$
- g** $\cos(2x - 18) = -0.7692$
 $2x - 18 = 180 - 39.715,$
 $180 + 39.715,$
 $39.715 - 180,$
 $-39.715 - 180$
 $= -219.715, -140.285,$
 $140.285, 219.715$
 $2x = -201.715, -122.285,$
 $158.285, 237.715$
 $x = -100.9, -61.1$
 $79.1, 118.9$
- h** $\sin 3x = -0.2941$
 $3x = 180 + 17.105,$
 $360 - 17.105,$
 $-17.105,$
 $17.105 - 180,$
 $-360 - 17.105,$
 $17.105 - 540$
 $= -522.895, -377.105,$
 $-162.895, -17.105,$
 $197.105, 342.895$
 $x = -174.3, -125.7, -54.3,$
 $-5.7, 65.7, 114.3$
- i** $\tan(2x + 135) = 1$
 $2x + 135 = 45, 180 + 45,$
 $360 + 45,$
 $45 - 180,$
 $= -135, 45,$
 $225, 405$
 $2x = -270, -90, 90, 270$
 $x = -135, -45, 45, 135$

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- 13** **a** $\operatorname{cosec}^2 \theta = 4$
 $\operatorname{cosec} \theta = \pm 2$
 $\sin \theta = \pm \frac{1}{2}$
 $\theta = 30^\circ, 180^\circ - 30^\circ \text{ or } 180^\circ + 30^\circ, 360^\circ - 30^\circ$
 $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
- b** $(\sec \theta + 1)(\sec \theta - 3) = 0$
 $\sec \theta = -1 \text{ or } 3$
 $\cos \theta = -1 \text{ or } \frac{1}{3}$
 $\theta = 180^\circ \text{ or } 70.5^\circ, 360^\circ - 70.5^\circ$
 $\theta = 70.5^\circ, 180^\circ, 289.5^\circ$
- c** $\cot \theta (\operatorname{cosec} \theta - 6) = 0$
 $\cot \theta = 0 \text{ or } \operatorname{cosec} \theta = 6$
 $\cos \theta = 0 \text{ or } \sin \theta = \frac{1}{6}$
 $\theta = 90^\circ, 360^\circ - 90^\circ \text{ or } 9.6^\circ, 180^\circ - 9.6^\circ$
 $\theta = 9.6^\circ, 90^\circ, 170.4^\circ, 270^\circ$
- d** $\frac{1}{\sin \theta} = \frac{4}{\cos \theta}$
 $\frac{\sin \theta}{\cos \theta} = \frac{1}{4}$
 $\tan \theta = \frac{1}{4}$
 $\theta = 14.0^\circ, 180^\circ + 14.0^\circ$
 $\theta = 14.0^\circ, 194.0^\circ$
- e** $2 \cos \theta = \frac{\cos \theta}{\sin \theta}$
 $2 \cos \theta \sin \theta = \cos \theta$
 $\cos \theta (2 \sin \theta - 1) = 0$
 $\cos \theta = 0 \text{ or } \sin \theta = \frac{1}{2}$
 $\theta = 90^\circ, 360^\circ - 90^\circ \text{ or } 30^\circ, 180^\circ - 30^\circ$
 $\theta = 30^\circ, 90^\circ, 150^\circ, 270^\circ$
- f** $5 \sin \theta - \frac{2}{\sin \theta} - 3 = 0$
 $5 \sin^2 \theta - 3 \sin \theta - 2 = 0$
 $(5 \sin \theta + 2)(\sin \theta - 1) = 0$
 $\sin \theta = -\frac{2}{5} \text{ or } 1$
 $\theta = 180^\circ + 23.6^\circ, 360^\circ - 23.6^\circ \text{ or } 90^\circ$
 $\theta = 90^\circ, 203.6^\circ, 336.4^\circ$
- 14** **a** $(2 \operatorname{cosec} x - 3)(\operatorname{cosec} x + 4) = 0$
 $\operatorname{cosec} x = -4 \text{ or } \frac{3}{2}$
 $\sin x = -\frac{1}{4} \text{ or } \frac{2}{3}$
 $x = -0.2527, -\pi + 0.2527 \text{ or } 0.7297, \pi - 0.7297$
 $x = -2.89, -0.25, 0.73, 2.41$
- b** $\frac{1}{\cos x} = \frac{3 \sin x}{\cos x}$
 $\sin x = \frac{1}{3}$
 $x = 0.3398, \pi - 0.3398$
 $x = 0.34, 2.80$
- c** $\frac{3}{\cos x} = \frac{2 \cos x}{\sin x}$
 $3 \sin x = 2 \cos^2 x$
 $3 \sin x = 2(1 - \sin^2 x)$
 $2 \sin^2 x + 3 \sin x - 2 = 0$
 $(2 \sin x - 1)(\sin x + 2) = 0$
 $\sin x = \frac{1}{2} \text{ or } -2 \text{ [no solutions]}$
 $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
 $x = 0.52, 2.62$
- d** $4 + \tan x - \frac{5}{\tan x} = 0$
 $\tan^2 x + 4 \tan x - 5 = 0$
 $(\tan x + 5)(\tan x - 1) = 0$
 $\tan x = -5 \text{ or } 1$
 $x = \pi - 1.3734, -1.3734 \text{ or } \frac{\pi}{4}, -\pi + \frac{\pi}{4}$
 $x = -2.36, -1.37, 0.79, 1.77$
- e** $\frac{1}{\sin x} = -\frac{5 \cos x}{\sin x}$
 $\cos x = -\frac{1}{5}$
 $x = \pi - 1.3694, -\pi + 1.3694$
 $x = -1.77, 1.77$
- f** $\frac{6 \sin x}{\cos x} = \frac{5}{\sin x}$
 $6 \sin^2 x = 5 \cos x$
 $6(1 - \cos^2 x) = 5 \cos x$
 $6 \cos^2 x + 5 \cos x - 6 = 0$
 $(3 \cos x - 2)(2 \cos x + 3) = 0$
 $\cos x = \frac{2}{3} \text{ or } -\frac{3}{2} \text{ [no solutions]}$
 $x = -0.84, 0.84$

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15 **a** LHS = $\frac{1}{\cos x} - \cos x$

$$\begin{aligned} &= \frac{1 - \cos^2 x}{\cos x} \\ &= \frac{\sin^2 x}{\cos x} \\ &= \sin x \times \frac{\sin x}{\cos x} \\ &= \sin x \tan x \\ &= \text{RHS} \end{aligned}$$

c LHS = $\frac{\sin x(\cot x - \cos x)}{\sin x(1 - \sin x)}$

$$\begin{aligned} &= \frac{\cos x - \sin x \cos x}{\sin x(1 - \sin x)} \\ &= \frac{\cos x(1 - \sin x)}{\sin x(1 - \sin x)} \\ &= \frac{\cos x}{\sin x} \\ &= \cot x \\ &= \text{RHS} \end{aligned}$$

b LHS = cosec $x - \cot x + \cot x - \cos x \cot x$

$$\begin{aligned} &= \frac{1}{\sin x} - \cos x \times \frac{\cos x}{\sin x} \\ &= \frac{1 - \cos^2 x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x} \\ &= \sin x \\ &= \text{RHS} \end{aligned}$$

d LHS = $\sin x \cos x + \sin x \cot x + \tan x \cos x + 1$

$$\begin{aligned} &= \sin x \cos x + \cos x + \sin x + 1 \\ &= \sin x (\cos x + 1) + \cos x + 1 \\ &= (\cos x + 1)(\sin x + 1) \\ &= \text{RHS} \end{aligned}$$

16 **a** $x = 0 \Rightarrow y = 2 - 3 - 5 = -6 \therefore (0, -6)$

b $y = 0 \Rightarrow 2 \cos x - \frac{3}{\cos x} - 5 = 0$

$$\begin{aligned} 2 \cos^2 x - 5 \cos x - 3 &= 0 \\ (2 \cos x + 1)(\cos x - 3) &= 0 \\ \cos x &= -\frac{1}{2} \text{ or } 3 \text{ [no solutions]} \\ x &= \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3} \\ x &= \frac{2\pi}{3}, \frac{4\pi}{3} \\ \therefore \left(\frac{2\pi}{3}, 0 \right) \text{ and } \left(\frac{4\pi}{3}, 0 \right) \end{aligned}$$