TRIGONOMETRY — *Answers*

1 $(2 \times 12.6) + 12.6\theta = 31.7$

$\theta = 6.5 \div 12.6 = 0.5159^c$

$A = \frac{1}{2} \times (12.6)^2 \times 0.5159 = 40.95 \text{ cm}^2$

2 **a** $\frac{1}{2} \times (7.3)^2 \times \theta = 38.4$

$\theta = 38.4 \div 26.645 = 1.44^c$ (3sf)

b chord $AB = 2 \times 7.3 \sin(\frac{1}{2}\theta) = 9.633$

arc $AB = 7.3\theta = 10.521$

$P = 9.633 + 10.521 = 20.2$ cm (3sf)

3 **a** $\frac{1}{2}r^2\theta = 40 \quad \therefore \ \theta = \frac{80}{r^2}$

$P = 2r + r\theta = 2r + (r \times \frac{80}{r^2})$

$= (2r + \frac{80}{r})$ cm

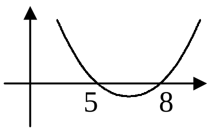
b $2r + \frac{80}{r} < 26$

$2r^2 + 80 < 26r$

$r^2 - 13r + 40 < 0$

$(r-5)(r-8) < 0$

$5 < r < 8$



4 **a** $AB^2 = 10^2 = 100$, $AC^2 + BC^2 = 6^2 + 8^2 = 100$

$AB^2 = AC^2 + BC^2$

$\therefore \ \angle ACB = 90°$ (converse of Pythagoras')

$\therefore$ triangle ABC is right-angled

b $\tan(\angle ABC) = \frac{AC}{BC} = \frac{3}{4} \quad \therefore \ \angle ABC = 0.64^c$

c $\angle BAC = \frac{\pi}{2} - 0.6435 = 0.9273$

area of sectors:

centre $A = \frac{1}{2} \times 4^2 \times 0.9273 = 7.4184$

centre $B = \frac{1}{2} \times 6^2 \times 0.6435 = 11.5830$

centre $C = \frac{1}{4} \times \pi \times 2^2 = 3.1416$

area of triangle $ABC = \frac{1}{2} \times AC \times BC = 24$

shaded area

$= 24 - (7.4184 + 11.5830 + 3.1416)$

$= 1.86 \text{ cm}^2$

5 **a** let centre of circle be O

let midpoint of AB be M

$AM^2 = OA^2 - OM^2 = 5^2 - 3^2 = 16$

$AM = 4 \quad \therefore \ AB = 8$ cm

b $\cos(\angle AOM) = \frac{3}{5}$

$\angle AOB = 2 \times \angle AOM = 1.8546^c$

arc $AB = 5 \times 1.8546 = 9.2730$

$P = 2 \times (6 + 14 - 8 + 9.2730) = 42.5$ cm

c area of segment

$= \frac{1}{2} \times 5^2 \times 1.8546 - \frac{1}{2} \times 5^2 \times \sin 1.8546^c$

$= 23.182 - 12 = 11.182$

area of logo $= (6 \times 14) + (2 \times 11.182)$

$= 106 \text{ cm}^2$ (3sf)

6 **a** $OC = (r + 2)$ cm

$A_1 = [\frac{1}{2} \times 8^2 \times \theta] - [\frac{1}{2} \times (r+2)^2 \times \theta]$

$= \frac{1}{2}\theta[64 - (r^2 + 4r + 4)]$

$= \frac{1}{2}\theta(60 - 4r - r^2) \text{ cm}^2$

b $A_2 = \frac{1}{2}r^2\theta$

$\therefore \ \frac{1}{2}\theta(60 - 4r - r^2) = 7 \times \frac{1}{2}r^2\theta$

$60 - 4r - r^2 = 7r^2$

$2r^2 + r - 15 = 0$

$(2r - 5)(r + 3) = 0$

$r > 0 \ \therefore \ r = 2.5$

7 let length of wire $= 3l$

area of $A = \frac{1}{2} \times l^2 \times \sin\frac{\pi}{3} = 0.43301l^2$

angle at centre of $B = l \div l = 1^c$

area of $B = \frac{1}{2} \times l^2 \times 1 = 0.5l^2$

% change $= \frac{0.5l^2 - 0.43301l^2}{0.43301l^2} \times 100\%$

$= 15.5\%$, increase