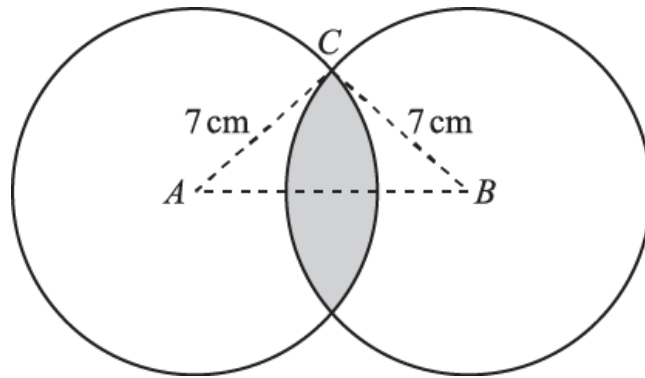


1.



The diagram shows two circles of radius 7 cm with centres  $A$  and  $B$ . The distance  $AB$  is 12 cm and the point  $C$  lies on both circles. The region common to both circles is shaded.

i. Show that angle  $CAB$  is 0.5411 radians, correct to 4 significant figures.

[2]

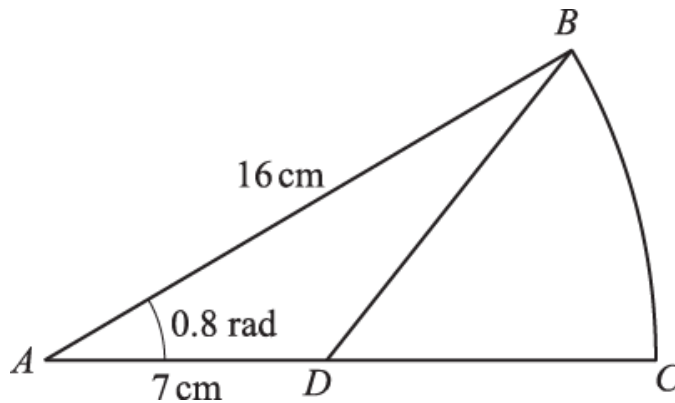
ii. Find the perimeter of the shaded region.

[2]

iii. Find the area of the shaded region.

[5]

2.



The diagram shows a sector  $BAC$  of a circle with centre  $A$  and radius 16 cm. The angle  $BAC$  is 0.8 radians. The length  $AD$  is 7 cm.

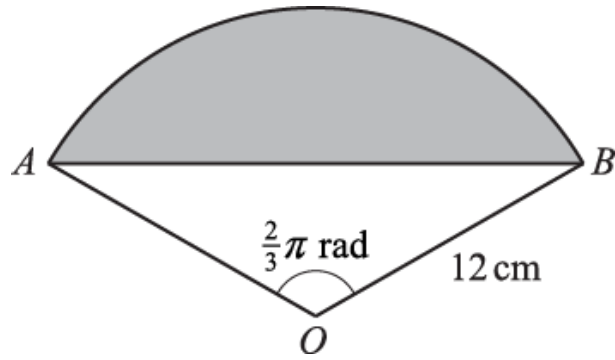
i. Find the area of the region  $BDC$ .

[4]

ii. Find the perimeter of the region  $BDC$ .

[4]

3.



The diagram shows a sector  $OAB$  of a circle, centre  $O$  and radius  $12\text{ cm}$ . The angle  $AOB$  is  $\frac{2}{3}\pi$  radians.

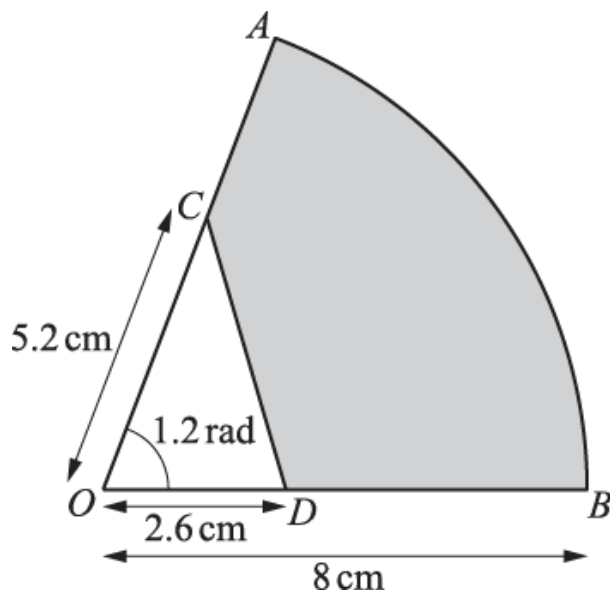
- i. Find the exact length of the arc  $AB$ .

[2]

- ii. Find the exact area of the shaded segment enclosed by the arc  $AB$  and the chord  $AB$ .

[5]

4.



The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius  $8\text{ cm}$ . The angle  $AOB$  is  $1.2$  radians. The points  $C$  and  $D$  lie on  $OA$  and  $OB$  respectively such that  $OC = 5.2\text{ cm}$  and  $OD = 2.6\text{ cm}$ .  $CD$  is a straight line.

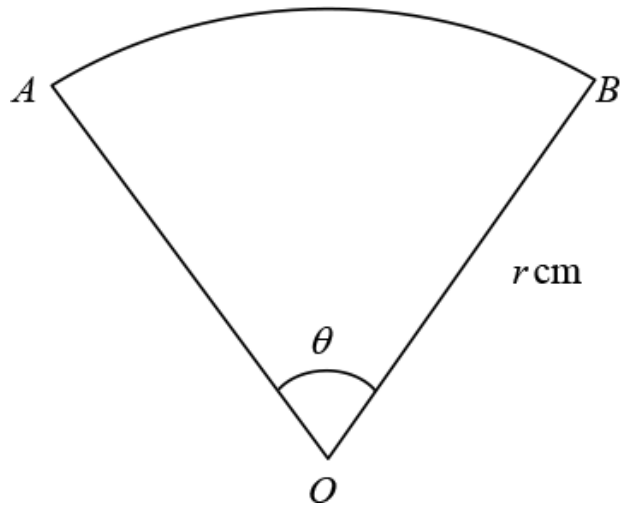
- i. Find the area of the shaded region  $ACDB$ .

[4]

- ii. Find the perimeter of the shaded region  $ACDB$ .

[5]

5.

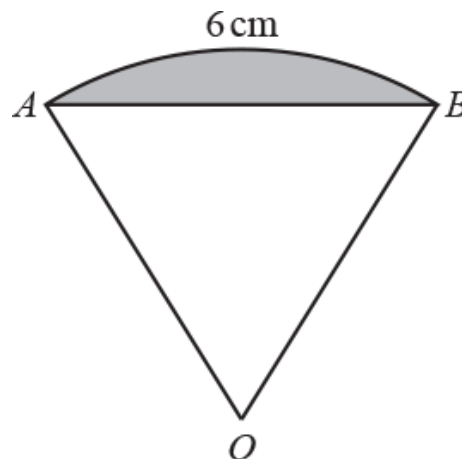


The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius  $r \text{ cm}$ . The angle  $AOB$  is  $\theta$  radians. The arc length  $AB$  is  $15 \text{ cm}$  and the area of the sector is  $45 \text{ cm}^2$ .

(a) Find the values of  $r$  and  $\theta$ . [4]

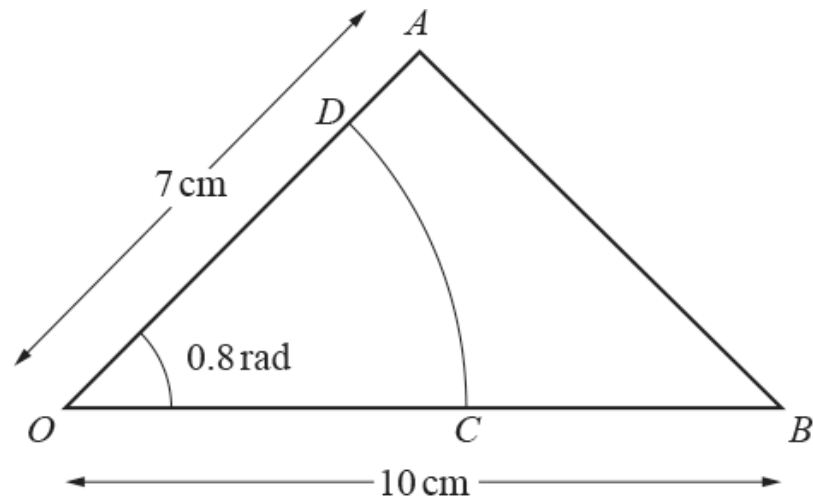
(b) Find the area of the segment bounded by the arc  $AB$  and the chord  $AB$ . [3]

6.



The diagram shows a sector  $AOB$  of a circle with centre  $O$ . The length of the arc  $AB$  is  $6 \text{ cm}$  and the area of the sector  $AOB$  is  $24 \text{ cm}^2$ . Find the area of the shaded segment enclosed by the arc  $AB$  and the chord  $AB$ , giving your answer correct to 3 significant figures. [6]

7.



The diagram shows the triangle  $AOB$ , in which angle  $AOB = 0.8$  radians,  $OA = 7\text{ cm}$  and  $OB = 10\text{ cm}$ .  $CD$  is the arc of a circle with centre  $O$  and radius  $OC$ . The area of the triangle  $AOB$  is twice the area of the sector  $COD$ .

(a) Find the length  $OC$ . [3]

(b) Find the perimeter of the region  $ABCD$ . [4]

END OF QUESTION paper

# Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance
1		i	$\cos^{-1} \frac{6}{7} = 0.5411$ <b>AG</b>	M1	Attempt correct method to find angle $CAB$
		i		A1	Obtain 0.5411
		i			
		ii	arc length = $7 \times (2 \times 0.5411) = 7.575$ perimeter = 15.2	M1	Attempt arc length using $7\theta$

Either use cosine rule or right-angled trigonometry  
Allow M1 for  $\cos A = \frac{6}{7}$  or equiv from cosine rule  
If first finding another angle, they must get as far as attempting angle  $CAB$  for the M1  
Allow in degrees or radians

Must be given to exactly 4sf, as per question  
If angle found as  $31^\circ$  then conversion to radians must be shown explicitly

**Examiner's Comments**

The majority of candidates were able to find the angle correctly, providing enough detail of the method used to be convincing. Using the cosine rule was a more popular approach than using a right-angled triangle. A surprising number of candidates first gave the required angle in degrees and then showed their conversion from degrees to radians, rather than simply setting their calculator into the appropriate mode.

Must be using  $r = 7$   
Allow if using  $\theta = 0.5411$  not 1.0822  
If no method shown then award M1 for value seen in the range [7.56, 7.58]  
M0 if using angle other than 0.5411 or 1.0822 (inc M0 for  $1.0822\pi$ ) but allow M1 if required angle is intended eg 0.54 or a slip when doubling 0.5411  
Allow valid method with degrees, but M0 for  $7\theta$  with  $\theta$  in degrees

		ii		A1	Obtain perimeter as 15.2, or better	<p>Allow equivalent method using fractions of the circle</p> <p>Allow 15.15, or anything that rounds to this with no errors seen</p> <p><b>Examiner's Comments</b></p> <p>Nearly all of the candidates could attempt to find the length of an arc, but not enough thought was given to deciding which angle to use. Most candidates did gain full marks, either by first doubling the given angle to get 1.0822 radians and then using this in the appropriate formula, or by using the original angle of 0.5411 radians and then multiplying the associated arc length by four.</p>
		ii				
		iii	<p>sector area = <math>\frac{1}{2} \times 7^2 \times (2 \times 0.5411) = 26.51</math></p> <p>triangle area = <math>\frac{1}{2} \times 7^2 \times \sin 1.082 = 21.63</math></p> <p>area of segment = 4.88</p> <p>shaded area = 9.76 cm<sup>2</sup></p>	M1*	Attempt area of one sector using $(\frac{1}{2}) \times 7^2 \times \theta$ , or equiv	<p>Allow if using <math>\theta = 0.5411</math> not 1.0822</p> <p>Allow M1 if 13.3 or 26.5 seen with no method</p> <p>M0 if using angle other than 0.5411 or 1.0822 (inc M0 for 1.0822<math>\pi</math>) unless clearly intended as correct angle</p> <p>Allow equivalent method using fractions of the circle</p> <p>Allow valid method with degrees, but M0 for <math>(\frac{1}{2})r^2\theta</math> with <math>\theta</math> in degrees</p> <p>Condone omission of <math>\frac{1}{2}</math>, but no other error</p> <p>May be seen explicitly or implied in method eg as part of <math>\frac{1}{2}r^2(\theta - \sin \theta)</math></p> <p>Condone omission of <math>\frac{1}{2}</math> from <math>\frac{1}{2}ab\sin\theta</math></p> <p>Allow if using <math>\theta = 0.5411</math> not 1.0822 in <math>(\frac{1}{2}) \times 7^2 \times \sin\theta</math></p> <p>Allow if attempting area of triangle <math>ABC</math></p> <p>Could be using radians or degrees</p> <p>Allow even if evaluated in incorrect mode</p> <p>If using a right-angled triangle, it must be <math>\frac{1}{2}bh</math>, and valid use of trig to find <math>b</math> or <math>h</math></p>
		iii		M1*	Attempt area of relevant triangle or area of rhombus	

	iii		A1	Obtain 4.88, or better, either as final answer or soi in method	<p>Could come from finding area of segment but omitting to double it</p> <p>Allow inaccuracy – values in range [4.85, 4.9]</p> <p>Allow even if value not seen explicitly – could be implied by part of a calculation or even by final answer</p> <p>Must be full and valid method – including attempted use of correct angle and subtraction in correct order</p> <p>Could find area of one segment and double it</p> <p>Other methods are possible eg 2 × sector minus rhombus</p> <p>Allow answer rounding to 9.76, no errors seen</p> <p><b><u>Examiner's Comments</u></b></p> <p>Most candidates were able to gain the first two marks on this question, one for attempting the area of a sector and one for attempting the area of a triangle, though this was not always relevant. Many candidates then made no further progress, usually because they had used the angle of 0.5411 incorrectly in <math>\frac{1}{2}r^2(\theta - \sin \theta)</math> or an equivalent method. Some candidates gave a little more thought to the method required, and a number of correct solutions were seen. The most common method was to find the area of a segment and then double it, and a rather more long-winded method was to first find the unshaded area in triangle <i>ABC</i>. An efficient and elegant solution was to find the difference between the total area of two sectors and the area of the rhombus, though some candidates mistakenly believed it to be a square. Some,</p>
	iii		M1d*	Attempt correct method to find required area	
	iii		A1	Obtain 9.76, or better	
	iii				

						otherwise correct, solutions were spoiled by a loss of accuracy in the working and hence in the final answer.
			<b>Total</b>	<b>9</b>		
2		i	sector area = $\frac{1}{2} \times 16^2 \times 0.8$ = 102.4	M1*	Attempt area of sector using $(\frac{1}{2}) r^2 \theta$ , or equiv	Condone omission of $\frac{1}{2}$ , but no other errors Must have $r = 16$ , not 7 M0 if $0.8\pi$ used not 0.8 M0 if $(\frac{1}{2}) r^2 \theta$ used with $\theta$ in degrees Allow equiv method using fractions of a circle
		i	triangle area = $\frac{1}{2} \times 16 \times 7 \times \sin 0.8$ = 40.2	M1*	Attempt area of triangle using $(\frac{1}{2}) ab \sin C$ or equiv	Condone omission of $\frac{1}{2}$ , but no other errors Angle could be in radians (0.8 not $0.8\pi$ ) or degrees ( $45.8^\circ$ ) Must have sides of 16 and 7 Allow even if evaluated in incorrect mode (gives 0.78) If using $\frac{1}{2} \times b \times h$ , then must be valid use of trig to find $b$ and $h$
		i		M1d*	Attempt area of sector – area of triangle  Obtain 62.2, or better	Using $\frac{1}{2} \times 16^2 \times (0.8 - \sin 0.8)$ will get M1 M0 M0
		i	area $BDC = 62.2 \text{ cm}^2$	A1	<b>Examiner's Comments</b>  This question was very well answered, with the majority scoring full marks. Candidates quoted the relevant formulae accurately, and then work to an acceptable degree of accuracy. As always, a few candidates felt the need to work in degrees rather than radians; not only is this more long-winded but using rounded values can affect the accuracy of the final answer. A few candidates decided that the region specified was itself a sector of a different circle and pursued this method, seeming not fazed when their two radii then appeared to be of different lengths.	Allow answers in range [62.20, 62.25] if > 3sf



		ii	$BD^2 = (16^2 + 7^2 - 2 \times 16 \times 7 \times \cos 0.8)$ $BD = 12.2$	M1	Attempt length of $BD$ using correct cosine rule	<p>Must be correct cosine rule</p> <p>Allow M1 if not square rooted, as long as <math>BD^2</math> seen</p> <p>M0 if <math>0.8\pi</math> used not 0.8</p> <p>Allow if evaluated in degree mode (gives 9.00)</p> <p>Allow if incorrectly evaluated – using <math>(16^2 + 7^2 - 2 \times 16 \times 7) \times \cos 0.8</math> gives 7.51</p> <p>Allow any equiv method, as long as valid use of trig</p> <p>Attempting the cosine rule in part (i) will only get credit if result appears in part (ii)</p>
		ii		A1	Obtain 12.2, or better	<p>Allow any answer rounding to 12.2, with no errors seen</p> <p>Could be implied in method rather than explicit</p>
		ii	$\text{arc } BC = 16 \times 0.8 = 12.8$	B1	State or imply that arc $BC$ is 12.8	<p>Allow if <math>16 \times 0.8</math> seen, even if incorrectly evaluated</p>
		ii	$\text{per} = 12.2 + 12.8 + 9 = 34.0 \text{ cm}$	A1	<p>Obtain 34, or better</p> <p><b>Examiner's Comments</b></p> <p>This was equally well done, with many concise and elegant solutions seen. The arc length was invariably correct, and candidates recognised the need to use the cosine rule though some struggled to evaluate this correctly, either by inserting imaginary brackets or by using the incorrect calculator mode. A surprising minority thought that <math>16 - 7 = 11</math>.</p>	<p>Accept 34 or 34.0, or any answer rounding to 34.0 if &gt; 3sf</p>
			<b>Total</b>	<b>8</b>		
3		i	$\text{arc} = 12 \times \frac{2\pi}{3}$	M1	Attempt use of $r\theta$	<p>Allow M1 if using <math>\theta</math> as <math>\frac{2}{3}</math></p> <p>M1 implied by sight of 25.1, or better</p> <p>M0 if <math>r\theta</math> used with <math>\theta</math> in degrees</p> <p>M1 for equiv method using fractions of a circle, with <math>\theta</math> as <math>120^\circ</math></p>

				Obtain $8\pi$ only		
		i	$= 8\pi$	A1	<p><b>Examiner's Comments</b></p> <p>Most candidates could quote the relevant formula and then obtain the correct length of the arc, though a number spoiled their answer by then giving the decimal approximation. Some candidates used more cumbersome methods involving fractions of a circle, which were usually correct though rarely resulted in an exact answer. A significant minority found the length of the chord <math>AB</math> instead, either through not reading the question carefully or through a lack of understanding of the terminology.</p>	Given as final answer – A0 if followed by 25.1
		ii	$\text{sector} = \frac{1}{2} \times 12^2 \times \frac{2\pi}{3} = 48\pi$	M1*	Obtain area of sector using $\frac{1}{2} r^2 \theta$	<p>Must be correct formula, including <math>\frac{1}{2}</math></p> <p>Must have <math>r = 12</math></p> <p>Allow M1 if using <math>\theta</math> as <math>\frac{2}{3}</math></p> <p>M0 if <math>\frac{1}{2} r^2 \theta</math> used with <math>\theta</math> in degrees</p> <p>M1 for equiv method using fractions of a circle, with <math>\theta</math> as <math>120^\circ</math></p> <p>M1 implied by sight of 151 or better</p>
		ii	$\text{triangle} = \frac{1}{2} \times 12^2 \times \sin \frac{2\pi}{3} = 36\sqrt{3}$	M1*	Attempt area of triangle using $\frac{1}{2} r^2 \sin \theta$	<p>Must be correct formula, including <math>\frac{1}{2}</math></p> <p>Must have <math>r = 12</math></p> <p>Allow M1 if using <math>\theta</math> as <math>\frac{2}{3}</math></p> <p>Allow even if evaluated in incorrect mode (2.63 or 41.8)</p> <p>If using <math>\frac{1}{2} \times b \times h</math>, then must be valid use of trig to find <math>b</math> and <math>h</math></p> <p>M1 implied by sight of 62.4, or better</p>
		ii	segment = $48\pi - 36\sqrt{3}$	M1d*	Correct method to find segment area	<p>Area of sector - area of triangle</p> <p>M0 if using <math>\theta</math> as <math>\frac{2}{3}</math></p> <p>Could be exact or decimal values</p>
		ii		A1	Obtain either $48\pi - 36\sqrt{3}$ or 88.4	<p>Allow decimal answer in range [88.44, 88.45] if &gt; 3sf</p>

		ii		A1	<p>Obtain <math>48\pi - 36\sqrt{3}</math> only</p> <p><b>Examiner's Comments</b></p> <p>Once again candidates could quote the relevant formulae and accurately substitute into them. It was common to see two separate exact values which then became a decimal once combined, possibly because of using a calculator. Candidates must appreciate that if an exact answer is requested then this should be the only final answer provided, and expect to get penalised if the decimal approximation is also given. The other common error was not ensuring that the calculator was in the correct mode when evaluating the area of the triangle.</p>	Given as final answer - A0 if followed by 88.4
			<b>Total</b>	<b>7</b>		
4		i	sector = $\frac{1}{2} \times 8^2 \times 1.2$ (= 38.4)	M1*	Attempt area of sector using $\frac{1}{2} r^2\theta$ , or equiv	<p>Must be correct formula, including <math>\frac{1}{2}</math></p> <p>M0 if <math>1.2\pi</math> used not 1.2</p> <p>M0 if <math>\frac{1}{2} r^2\theta</math> used with <math>\theta</math> in degrees</p> <p>Allow equiv method using fractions of a circle</p>
		i	$\frac{1}{2} \times 2.6 \times 5.2 \times \sin 1.2$ (= 6.3)	M1*	Attempt area of triangle using $\frac{1}{2} ab \sin C$ or equiv	<p>Must be correct formula, including <math>\frac{1}{2}</math></p> <p>Angle could be in radians (1.2 not <math>12\pi</math>) or degrees (<math>68.8^\circ</math>)</p> <p>Must have sides of 2.6 and 5.2</p> <p>Allow even if evaluated in incorrect mode</p> <p>If using <math>\frac{1}{2} \times b \times h</math>, then must be valid use of trig to find <math>b</math> and <math>h</math></p>
		i	$38.4 - 6.3 = 32.1$	M1d*	Attempt area of sector – area of triangle	<p>Using <math>\frac{1}{2} \times 8^2 \times (1.2 - \sin 1.2)</math> will get M1 M0 M0</p> <p>Need area of sector &gt; area of triangle</p>
		i		A1	<p>Obtain 32.1, or better</p> <p><b>Examiner's Comments</b></p> <p>This question was very well-answered, and the vast</p>	Allow final answers rounding to 32.10 if > 3sf

					majority of candidates gained all of the marks available. The most efficient method was to work in radians throughout, though there were inevitably candidates who decided to use an equivalent method involving degrees instead. As long as there was no subsequent loss of accuracy, then this method was accepted. A few candidates simply assumed that triangle $OCD$ was isosceles, and the other common error was to use the formula for the area of a segment rather than considering carefully the question posed.	
	ii	$8 \times 1.2 = 9.6$		M1*	Attempt use of $r\theta$ , or equiv	Allow if $8 \times 1.2$ seen, even if incorrectly evaluated
	ii	$CD^2 =$ $2.6^2 + 5.2^2 - 2 \times 2.6 \times 5.2 \times \cos 1.2$		M1*	Attempt use of cosine rule, or equiv, to find $CD$	Must be correct cosine rule Allow M1 if not square rooted, as long as $CD^2$ seen MO if $1.2\pi$ used not 1.2 Allow if incorrectly evaluated, inc mode Allow any equiv method, as long as valid use of trig
	ii	$CD = 4.90$ or $\sqrt{24}$		A1	Obtain $CD = 4.90$ or $\sqrt{24}$	Allow any answer in range [4.89, 4.90], with no errors seen Could be implied in method rather than explicit
	ii	perimeter = $2.8 + 4.9 + 5.4 + 9.6$		M1d*	Attempt perimeter of region  Obtain 22.7, or better	$(8 - 5.2) + (8 - 2.6) +$ their $AB$ + their $CD$ (not their $CD^2$ )
	ii	= 22.7		A1	<u>Examiner's Comments</u>  Candidates struggled a little more with this part of the question, but it was still very well-answered. Once again, many efficient and effective solutions were seen. Most candidates gained the first mark for finding correctly the required arc length, and could then attempt the cosine rule. Given that this rule is given in the formula book, it	Accept any answer in range [22.69, 22.70] if > 3sf

					was disappointing to see errors when it was initially quoted. Some candidates made errors when evaluating the expression, and these included treating $(b^2 + c^2 - 2bc)$ as the coefficient of $\cos A$ and omitting to square root the evaluated expression. There was also a loss of accuracy in some solutions from premature approximation, and candidates would be well-advised to make efficient use of their calculator and only round when giving their final answer.
			<b>Total</b>	<b>9</b>	
5	a	$r\theta = 15$ $\frac{1}{2}r^2\theta = 45$ $\frac{1}{2}r(15) = 45$ $r = 6$ and $\theta = 2.5$	<b>B1(AO1.1)</b> <b>B1(AO1.1)</b> <b>M1(AO3.1a)</b>  <b>A1(AO1.1)</b> <b>[4]</b>	<div style="border: 1px solid black; padding: 5px;"> Accept any method for solving the equations simultaneously </div>	
	b	$\frac{1}{2}(6)^2 \sin\left(\frac{5}{2}\right)$ $45 - \text{their } \frac{1}{2}(6)^2 \sin\left(\frac{5}{2}\right)$  34.2(cm <sup>2</sup> )	<b>B1FT(AO1.1)</b>  <b>M1(AO1.1)</b> <b>A1FT(AO1.1)</b>  <b>[3]</b>	<div style="border: 1px solid black; padding: 5px;"> FT their <math>r</math> and <math>\theta</math>        FT their <math>r</math> and <math>\theta</math> </div>	
			<b>Total</b>	<b>7</b>	



				<p>Obtain <math>r = 8</math>, <math>\theta = 0.75</math> (aef)</p> <p>Attempt area of segment</p> <p>Obtain 2.19, or better</p>	<p>and <math>\theta</math></p> <p>Both values required</p> <p>24 – area of triangle, using <math>\frac{1}{2}r^2\sin\theta</math> or equiv Allow if evaluated in degree mode (gives 23.58) Allow M1 for attempting <math>\frac{1}{2}r^2(\theta - \sin\theta)</math> with their <math>r</math> and <math>\theta</math>, even if this does not give area of sector as 24</p> <p>Allow final answer in range [2.187, 2.188] if &gt; 3sf</p> <p>Could use variables other than <math>r</math> and <math>\theta</math></p> <p><b>Alt method for working in degrees</b> B1 - state</p>	
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A1

6

$$\frac{\theta}{360} \times 2\pi r = 6$$

B1 - state

$$\frac{\theta}{360} \times \pi r^2 = 24$$

M1 - attempt to solve

simultaneously

A1 - obtain  $r = 8$ ,

$\theta = 43.0^\circ$  or better (42.97...)

M1 - attempt area of segment  
NB using  $\frac{1}{2}r^2(\theta - \sin\theta)$  with  $\theta$  in degrees is M0

as incorrect attempt at area of sector

A1 - obtain 2.19 or better

#### Examiner's Comments

This question was also very well answered, with many fully correct solutions being seen. Candidates coped well with the lack of structure in this question and were able to formulate an effective method to find the area of the segment. Most candidates could state two relevant equations, either by quoting the relevant formulae or by working in fractions of a circle. An uncertainty over terminology resulted in some candidates using 24 as the area of the triangle rather than the sector. Having obtained two correct equations, most candidates were able to solve them simultaneously although a significant minority were



					unable to simplify the indices involved. Candidates were familiar with the process for finding the area of a segment and many gained credit for doing so, even if there had been errors earlier on. A minority of candidates elected to work in degrees rather than radians throughout. A number managed to do so successfully, but the awkwardness of the numbers involved proved too much for others. Whilst there is nothing wrong with this approach, candidates should always attempt to use the most efficient method.	
			<b>Total</b>	<b>6</b>		
7	a	$0.5 \times 10 \times 7 \times \sin 0.8 = 25.107$  $0.5 \times r^2 \times 0.8 = 12.55$  $r = 5.6 \text{ cm}$	<b>M1 (AO 3.1a)</b>  <b>M1 (AO 1.1)</b>  <b>A1 (AO 1.1)</b>  <b>[3]</b>	Attempt area of triangle  Equate area of sector to half of area of triangle and attempt to find $r$  Obtain correct value for $r$	5.6 or better (5.60217...)	
	b	$CD = 5.6 \times 0.8 = 4.48$  $AB = \sqrt{7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 0.8} = 7.17$  $ABCD = 7.17 + (10 - 5.6) + 4.48 + (7 - 5.6)$  =17.5 cm	<b>M1 (AO 3.1a)</b>  <b>M1 (AO 1.1)</b>  <b>M1 (AO 1.1)</b>  <b>A1 (AO 1.1)</b>  <b>[4]</b>	Attempt arc length using $r\theta$  Attempt $AB$ using cosine rule  Attempt		

					<table border="1"> <tr> <td>perimeter of <i>ABCD</i></td> <td>Allow 17.4 www</td> </tr> <tr> <td>Obtain 17.5 cm</td> <td></td> </tr> </table>	perimeter of <i>ABCD</i>	Allow 17.4 www	Obtain 17.5 cm		
perimeter of <i>ABCD</i>	Allow 17.4 www									
Obtain 17.5 cm										
			Total	7						