1. i. Prove that

$$\cos^2(\theta + 45°) - \tfrac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2\theta.$$

[4]

ii. Hence solve the equation

$$6\cos^2(\tfrac{1}{2}\theta + 45°) - 3(\cos\theta - \sin\theta) = 2$$

for $-90° < \theta < 90°$.

[3]

iii. It is given that there are two values of θ, where $-90° < \theta < 90°$, satisfying the equation

$$6\cos^2(\tfrac{1}{3}\theta + 45°) - 3(\cos\tfrac{2}{3}\theta - \sin\tfrac{2}{3}\theta) = k,$$

where *k* is a constant. Find the set of possible values of *k*.

[3]

2. It is given that $f(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

i. Show that $f(\theta) = \cos\theta$. Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8\cos^4\theta - 3.$$

[6]

ii. Hence

a. determine the greatest and least values of $\dfrac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies,

[3]

b. solve the equation

$$\sin(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$.

[4]

3. Prove that $\sin^2(\theta + 45)° - \cos^2(\theta + 45)° \equiv \sin 2\theta°$. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	State cosθ cos 45 – sinθ sin 45	B1	or equiv including use of decimal approximation for $\frac{1}{\sqrt{2}}$	
		i	Use correct identity for sin 2θ or cos 2θ	B1	must be used; not earned for just a separate statement	
		i	Attempt complete simplification of left-hand side	M1	with relevant identities but allowing sign errors, and showing two terms involving sinθ cosθ	
		i	Obtain $\sin^2 θ$	A1	AG; necessary detail needed **Examiner's Comments** 24% of the candidates earned all four marks for this proof. Success needed thorough knowledge of the relevant trigonometry plus care with detail. Most candidates earned a mark for an appropriate identity involving sin2θ or cos2θ but, for many candidates, that was the only mark they earned. The main problem concerned the term $\cos^2 (θ + 45°)$; candidates realised that expansion was needed but, all too often, this was $\cos^2 θ\cos^2 45° – \sin^2 45°$. Other candidates did start with $(\cos θ \cos 45° – \sin θ \sin 45°)^2$, or the corresponding version involving the exact values of cos 45° and sin 45°, but the expansion then often omitted the term involving sin θ cos θ. Once mistakes like these had been made, the identity could not be proved but many candidates did persevere with some involved, if doomed, attempts to try to reach $\sin^2θ$. A few candidates showed pleasing mathematical awareness by rewriting the first term of the left-hand side as $\frac{1}{2}+\frac{1}{2}\cos(2\theta+90°)$ which simplifies to $\frac{1}{2}-\frac{1}{2}\sin 2\theta$; completion of the proof followed quickly.	

		ii	Use identity to produce equation of form $\sin\frac{1}{2}\theta = c$	M1	condoning single value of constant *c* here (including values outside the range −1 to 1); M0 for sin Θ = *c* unless value(s) are subsequently doubled	
		ii	Obtain 70.5 or 70.6	A1	or greater accuracy 70.528…	
		ii	Obtain −70.5 or −70.6	A1√	or greater accuracy −70.528…; following first answer; and no other answer between −90 and 90; answer(s) only: 0/3 **Examiner's Comments** Many candidates seemed unaware of the fact that the identity established in part (i) was related to the equation in this part and they tried manipulation of the given equation. Others did try to exploit the earlier part but, with a lack of attention to detail, proceeded to solve $6\sin^2\theta = 2$. Many candidates did proceed with the correct $6\sin^2\frac{1}{2}\theta = 2$ and 25% of all the candidates concluded with the two correct angles. Other candidates gave one correct answer 70.5° but omitted the other possibility of 70.5–°.	
		iii	State or imply $6\sin^2\frac{1}{3}\theta = k$	B1		
		iii	Attempt to relate *k* to at least $6\sin^2 30°$	M1		
		iii	Obtain $0 < k < \frac{3}{2}$	A1	condone use of ≤ **Examiner's Comments** This was a testing conclusion to the paper but 6% of candidates were equal to the challenge and answered accurately. There were many others who made significant progress, recognising that the value of *k* was less than $\frac{3}{2}$ but then concluding with $-\frac{3}{2} < k < \frac{3}{2}$, a result that overlooks the fact that $6\sin^2\frac{1}{3}\theta$ cannot be negative.	
			Total	10		

2		i	Use at least one addition formula accurately	M1	Without substituting values for cos30°, etc. yet	
		i	Obtain $\cos\theta$	A1	AG; necessary detail needed	
		i	State $\cos 4\theta = 2\cos^2 2\theta - 1$	B1	Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$	
		i	Attempt correct use of relevant formulae to express in terms of $\cos\theta$	M1	Or in terms of $\cos\theta$ and $\sin\theta$	
		i	Obtain correct unsimplified expression in terms of $\cos\theta$ only	A1	e.g. $2(2c^2 - 1)^2 - 1 + 4(2c^2 - 1)$	
		i	Simplify to confirm $8\cos^4\theta - 3$	A1	AG; necessary detail needed	
		i			**Examiner's Comments** This question contained challenges for even the best candidates and only 13% of the candidates recorded all thirteen marks. The first two marks of part (i) were obtained by most but convincing and concise responses to the subsequent proof were not so common. Many candidates did not take the trouble to present solutions in such a way that they were easy to follow, or indeed to read. On some scripts, it was often difficult for examiners to decide whether candidates had written $\cos2\theta$ or $\cos^2\theta$. In other cases, parts of the proof were scattered around the page and efforts to reassemble the parts did not always succeed. The main difficulty was dealing with $\cos4\theta$. Some decided that, since $\cos2\theta = \cos^2\theta - \sin^2\theta$, $\cos4\theta$ must be $\cos^4\theta - \sin^4\theta$. Many did state $\cos4\theta = \cos^2 2\theta - \sin^2 2\theta$ but use of this did lead to involved expressions involving $\cos\theta$ and $\sin\theta$; considerable care was then needed to reach a successful conclusion. The best solutions usually involved use of $\cos4\theta = 2\cos^2 2\theta - 1$ and $\cos2\theta = 2\cos^2\theta - 1$.	
		ii	(a) Obtain $\frac{1}{12}$	B1		
		ii	Substitute 0 for $\cos\theta$ in correct expression	M1	No need to specify greatest and least	
		ii	Obtain $\frac{1}{4}$	A1		

		ii			**Examiner's Comments** Part (ii)(a) proved demanding for many; about as many earned no marks as earned all three. A few carelessly considered $\frac{1}{8\cos^4\theta - 3}$. For those dealing with the correct $\frac{1}{8\cos^4\theta + 4}$, the value $\frac{1}{12}$ usually appeared but many candidates mistakenly decided that the other requested value would result from $\cos^4\theta$ being −1.	
		ii	**(b)** State or imply $8\cos^4(3\alpha) - 3 = 1$	B1	Or $2\cos^2 6\alpha + 4\cos 6\alpha - 2 = 0$	
		ii	Attempt correct method to obtain at least one value of α	M1	Allow for equation of form $\cos^4(3\alpha) = k$ where $0 < k < 1$ or for three-term quadratic equation in cos6α	
		ii	Obtain 10.9	A1	Or greater accuracy 10.921…	Answer(s) only: 0/4
		ii	Obtain 49.1	A1	Or greater accuracy 49.078…; and no others between 0 and 60	
		ii			**Examiner's Comments** Many candidates saw no connection between the equation in part (ii)(b) and the results in part (i). Their attempts involved starting afresh and it was very seldom that any significant progress was made. Some made a connection with the first result from part (i) and formed the equation cos12α + 4cos6α = 1. Not all knew how to deal with this; for those who did, replacement of 6α by another letter sometimes meant that the solution of the equation was not completed correctly. The other successful approach involved recognising the link with the main result from part (i). However, the attempt to solve the corresponding equation $\cos^4(3\alpha) = \frac{1}{2}$ frequently led to only one value of a as candidates omitted the value	

					corresponding to $\cos(3\alpha) = -\sqrt[4]{\frac{1}{2}}$.	
			Total	13		
3			<u>Summary of method</u> Use of $\cos(A + B)$ or $\sin(A + B)$ or $\cos 2\theta$ formula Correct result Use of one of the above or $\sin 2\theta$ formula Correctly obtain result <u>Example of method</u> $\sin^2(\theta + 45) - \cos^2(\theta + 45) \equiv -\cos 2(\theta + 45)$ $\equiv -\cos(2\theta + 90)$ $\equiv -[\cos 2\theta \cos 90 - \sin 2\theta \sin 90] \equiv \sin 2\theta$ AG	M1 (AO 3.1a) A1 (AO 2.1) M1 (AO 1.1) A1 (AO 1.1) M1 A1 M1 A1 [4]	Correct formula Correct formula Use of correct $\cos 2\theta$ formula Correct result Use of correct $\cos(A + B)$ formula Must see this step and final answer **<u>Examiner's Comments</u>** A large variety of correct methods were seen, some shorter than others. Some candidates made mistakes in quoting formulae, despite the formulae being given on the question paper.	
			Total	4		