1. Solve the equation $2\sec^2\theta = 5\tan\theta$, for $0 \le \theta \le \pi$.

[6]

2. Show that the equation $\operatorname{cosec} x + 5\cot x = 3\sin x$ may be rearranged as

$$3\cos^2 x + 5\cos x - 2 = 0.$$

Hence solve the equation for $0° \leqslant x \leqslant 360°$, giving your answers to 1 decimal place.

[7]

3. **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta + 2\tan\theta = 4$ for $0° \le \theta < 360°$. [4]

4. **In this question you must show detailed reasoning.**

(a) Prove that $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(b) Hence solve the equation $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1}{3}$ for $0° < \theta < 360°$. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1			$2\sec^2\theta = 5\tan\theta$	6	
			$\Rightarrow 2(1 + \tan^2\theta) = 5\tan\theta$	M1	$\sec^2\theta = 1 + \tan^2\theta$ **used**
			$\Rightarrow 2\tan^2\theta - 5\tan\theta + 2 = 0$	A1	correct quadratic **oe**
			$\Rightarrow (2\tan\theta - 1)(\tan\theta - 2) = 0$	M1	solving their quadratic for $\tan\theta$ (follow rules for solving as in Question 1 [*,*]
			$\Rightarrow \tan\theta = ½$ or 2	A1	www
			$\Rightarrow \theta = 0.464$,	A1	first correct solution (or better)
			1.107	A1	second correct solution (or better) and no others in the range Ignore solutions outside the range. **SC** A1 for both 0.46 and 1.11 **SC** A1 for both 26.6° and 63.4° (or better) Do not award **SC**s if there are extra solutions in range.
			.... OR		....
			$2/\cos^2\theta = 5\sin\theta/\cos\theta$	M1	**using** both sec = 1/cos and tan = sin/cos
			$\Rightarrow 2\cos\theta = 5\sin\theta\cos^2\theta$, $\cos\theta \neq 0$	A1	correct one line equation $2 - 5\sin\theta\cos\theta = 0$ or $2\cos\theta = 5\sin\theta\cos^2\theta$ oe (or common denominator). Do not need $\cos\theta \neq 0$ at this stage.
			$\Rightarrow \cos\theta(2 - 5\sin\theta\cos\theta)\ 0$ $\Rightarrow \cos\theta = 0$, *or* $\sin 2\theta = 0.8$	M1	**using** $\sin 2\theta = 2\sin\theta\cos\theta$ oe eg $2 = 5\sin\theta\sqrt{(1 - \sin^2\theta)}$ and squaring
			$\Rightarrow \sin 2\theta = 0.8$	A1	$\sin2\theta = 0.8$ or, say, $25\sin^4\theta - 25\sin^2\theta + 4 = 0$

			$\Rightarrow 2\theta = 0.9273$ *or* 2.2143 $\Rightarrow \theta = 0.464$	A1	first correct solution (or better)
			1.107	A1	second correct solution (or better) and no others in range Ignore solutions outside the range SCs as above **Examiner's Comments** Candidates seemed equally to choose the two approaches in the mark scheme to solve the trigonometric equation. Both were equally successful and few offered extra unnecessary solutions. The main error was to give insufficient accuracy in the final solutions. Where solving tan θ=2 in degrees leads to θ=63.4° to 3sf, giving θ= 1.11 radians = 63.598° (63.6°) and θ=1.1radians=63.0° were insufficiently accurate so we needed θ=1.107radians to achieve the same accuracy as 63.4°.
			Total	6	
2			cosec x + 5 cot x = 3 sin x $\Rightarrow \dfrac{1}{\sin x} + \dfrac{5\cos x}{\sin x} = 3\sin x$	M1	**using** cosec x = 1/sin x and cot x = cos x / sin x
			$\Rightarrow 1 + 5\cos x = 3\sin^2 x = 3(1 - \cos^2 x)$	M1	$\cos^2 x + \sin^2 x = 1$ **used** (both M marks must be part of same solution in order to score both marks)
			$\Rightarrow 3\cos^2 x + 5\cos x - 2 = 0$ *	A1	**AG** (**Accept working backwards, with same stages needed**)
			$\Rightarrow (3\cos x - 1)(\cos x + 2) = 0$	M1	use of correct quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving correct coeffs 3 and −2 when multiplied out) or comp square oe
			$\Rightarrow \cos x = 1/3$,	A1	cos x = 1/3 www
			$x = 70.5°$,	A1	for 70.5° or first correct solution, condone over-specification (ie 70.5° or better eg 70.53°, 70.5288° etc),

			289.5°	A1	for 289.5° or second correct solution (condone over-specification) and no others in the range Ignore solutions outside the range **SC**A1A0 for incorrect answers that round to 70.5 and 360-their ans, eg 70.52 and 289.48 **SC** Award A1A0 for 1.2, 5.1 radians (or better) Do not award SC marks if there are extra solutions in the range <u>Examiner's Comments</u> Many candidates scored full marks when showing that the trigonometric equation could be rearranged as a quadratic and then solving it. Where there were errors, these were usually in the first part when trying to establish the given result. Errors included failing to use the correct trigonometric identities, failing to use $\sin^2\theta + \cos^2\theta = 1$ or squaring the original expression term by term. Few candidates would say $x+3=7$ so $x^2+9=49$ and yet they happily square cosec x+5cot x=3sin x term by term. Those who were unable to complete the first part sensibly then proceeded to solve the quadratic equation. Few errors were seen here. Occasionally the final solution was incorrect and few candidates offered additional incorrect solutions.
			Total	7	
3			$(1 + \tan^2\theta) + 2\tan\theta = 4$ $\tan^2\theta + 2\tan\theta - 3 = 0$ $(\tan\theta - 1)(\tan\theta + 3) = 0$ When $\tan\theta - 1$, $\theta = 45^\circ, 225^\circ$	M1 (AO 3.1a) M1 (AO 1.1a) A1 (AO 1.1b) A1 (AO 1.1b)	DR Using appropriate trig identity Showing algebraic method for solving their quadratic Must attempt to reach an equation with only one trig function eg $20\cos^4\theta - 12\cos^2\theta + 1 = 0$ Or $\sqrt{5}\sin(2\theta - 63.4^\circ) = 1$

			When tan $\theta = -3$, $\theta = 108.4^\circ$, 288.4°	[4]	Any two correct values for θ All correct values for θ and no extras in the interval. Ignore values outside the required interval. **Examiner's Comments** Candidates who used the identity $\sec^2\theta = 1 + \tan^2\theta$ generally went on to obtain most of the marks. Only a few candidates tried to rewrite the equation in terms of $\cos\theta$ as this is a much more difficult method requiring both sides to be squared and spurious solutions eliminated. Candidates did not get far enough into this method to obtain the method mark.
			Total	4	
4		a	$$(\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$ $$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$	M1 (AO 2.1) M1 (AO 2.1) A1 (AO 2.1) [3]	Using $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$ Using $\sin^2\theta = 1 -$

			$=\dfrac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}=\dfrac{1-\cos\theta}{1+\cos\theta}$		$\cos^2\theta$ **AG** Factorising must be shown	
		b	$\dfrac{1-\cos\theta}{1+\cos\theta}=\dfrac{1}{3}\Rightarrow 3-3\cos\theta=1+\cos\theta\Rightarrow\cos\theta=\frac{1}{2}$ $\theta = 60°, 300°$	M1 (AO 1.1a) A1 (AO 1.1) A1 (AO 1.1) [3]	Attempt to rearrange and find $\cos\theta$ For one correct value for θ For second correct value; do not allow if additional values in range given, but ignore values outside range	
			Total	6		