1.

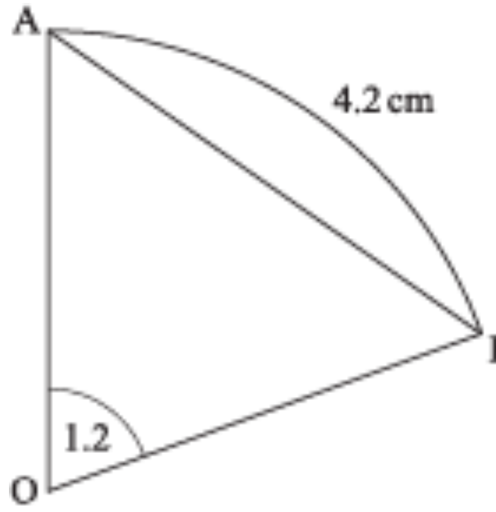


Fig. 4

Fig. 4 shows sector OAB with sector angle 1.2 radians and arc length 4.2 cm. It also shows chord AB.

i. Find the radius of this sector.

[2]

ii. Calculate the perpendicular distance of the chord AB from O.

[2]

2.

i. Starting with an equilateral triangle, prove that $\cos 30° = \frac{\sqrt{3}}{2}$.

[2]

ii. Solve the equation $2 \sin\theta = -1$ for $0 \leqslant \theta \leqslant 2\pi$, giving your answers in terms of π.

[3]

3.

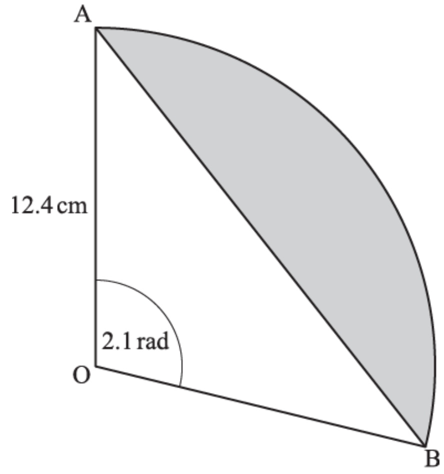


Fig. 6

A circle with centre O has radius 12.4 cm. A segment of the circle is shown shaded in Fig. 6. The segment is bounded by the arc AB and the chord AB, where the angle AOB is 2.1 radians. Calculate the area of the segment.

[4]

4. Show that the equation $\sin^2 x = 3\cos x - 2$ can be expressed as a quadratic equation in $\cos x$ and hence solve the equation for values of 3^x between 0 and 2π.

[5]

5. A sector of a circle has angle 1.5 radians and area 27 cm^2. Find the perimeter of the sector.

[4]

6. A sector of a circle has radius r cm and sector angle θ radians. It is divided into two regions, A and B. Region A is an isosceles triangle with the equal sides being of length a cm, as shown in Fig. 6.

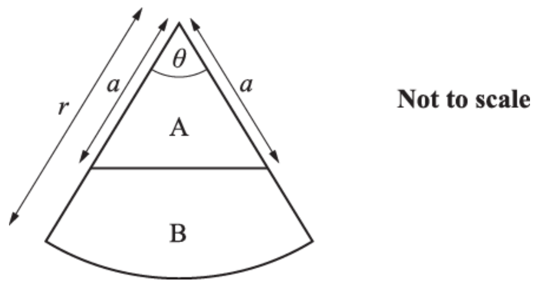


Fig. 6

i. Express the area of B in terms of a, r and θ.

[2]

ii. Given that $r = 12$ and $\theta = 0.8$, find the value of a for which the areas of A and B are equal. Give your answer correct to 3 significant figures.

[2]

7. Fig. 1 shows a sector of a circle of radius 7 cm. The area of the sector is 5 cm^2.

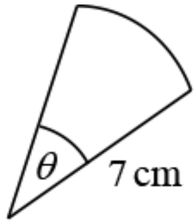


Fig. 1

Find the angle θ in radians [2]

8. Fig. 7 shows a circle with centre A and radius 6 cm, and a circle with centre B and radius 2 cm. The two circles touch. DCE is a common tangent to the circles. ABE is a straight line.

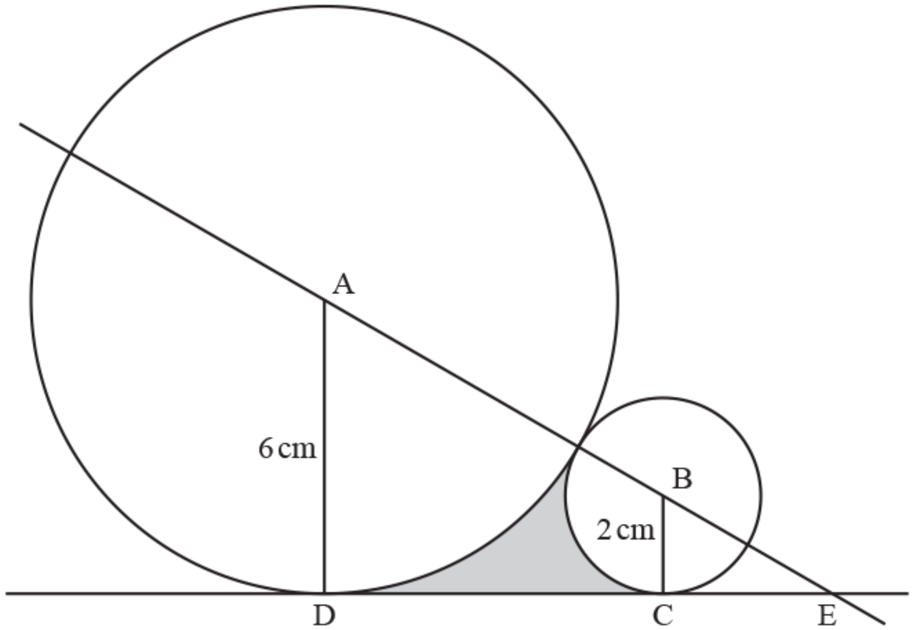


Fig. 7

(a) Give a reason why angles ADE and BCE are right angles. [1]

(b) Show that angle DAB is $\frac{1}{3}\pi$ radians. [2]

(c) Show that trapezium ABCD has area $16\sqrt{3}$ cm^2. [4]

(d) Calculate the area of the shaded region enclosed by the two circles and the common tangent DCE. Give your answer correct to 3 decimal places. [3]

9. Fig. 3 shows a circle with centre O and radius 1 unit. Points A and B lie on the circle with angle AOB = θ radians. C lies on AO, and BC is perpendicular to AO.

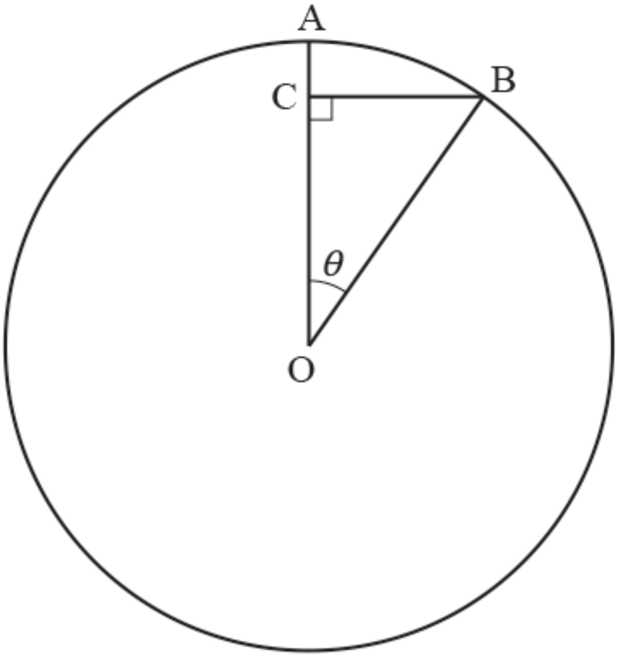


Fig. 3

Show that, when θ is small, $\mathrm{AC} \approx \frac{1}{2}\theta^2$. [2]

10. Fig. 5 shows triangle ABC where AB = 7 cm, BC = 8 cm and AC = 5 cm. The curve is an arc of a circle with centre C and radius r cm.

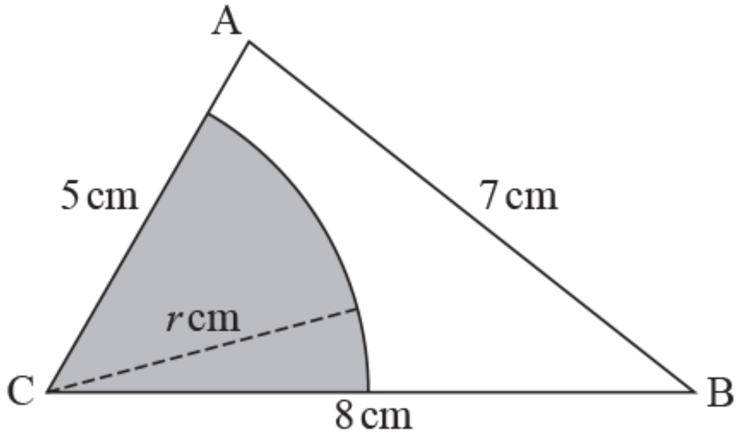


Fig. 5

Exactly half the area of the triangle is shaded. Find the value of r. [5]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$1.2r = 4.2$	M1	or $\frac{68.7549...}{360} \times 2\pi r = 4.2$ with θ to 3 sf or better	**B2** if correct answer unsupported
		i	3.5 cao	A1	**Examiner's Comments** Almost all candidates achieved full marks on this question. Some converted to degrees and rounded prematurely, thus losing the accuracy mark for the final answer, and a few used the formula for the area of a sector.	
		ii	$\cos 0.6 = \frac{d}{\text{their}3.5}$	M1	or $\cos 34.377.. = \frac{d}{\text{their}3.5}$ with θ to 3 sf or better	or correct use of Sine Rule with 0.9708(55.623°) or area = 5.709 = 0.5 × h × 3.952, or $3.5^2 - 1.976^2 = d^2$
		ii	2.888.. to 2.9	A1	**Examiner's Comments** This straightforward question defeated a surprisingly large number of candidates. Many of these misunderstood the question and used the Cosine Rule to calculate the length AB, or simply answered their own question and calculated the area of the sector or the segment. Many of the successful candidates used convoluted methods, such as finding AB and then using Pythagoras — premature rounding sometimes caused a mark to be lost; forgetting to halve AB cost both marks. The Sine Rule was sometimes used successfully — but this was sometimes spoiled by the use of sinπ in conjunction with	

					3.5. A few candidates found the area of the triangle and then used ½ base × height. Surprisingly few were able to use the expected approach: $d = 3.5\cos 0.6$.	
			Total	4		
2		i	clear diagram or explanation starting with equilateral triangle correctly showing 30 as half angle and sides 1 and 2 or multiples of these lengths	B1		units for sides and angle not required
		i	correct use of Pythagoras *and* adjacent and hypotenuse correctly identified to obtain $\cos 30° = \frac{\sqrt{3}}{2}$ given result	B1	adjacent and hypotenuse may be identified on diagram Examiner's Comments Over half of candidates failed to score on this question. A surprising number drew "equilateral" triangles with unequal angles or sides, defined the cosine ratio incorrectly or not at all, or were unable to use Pythagoras correctly to obtain the third side of their right angled triangle. Generally, candidates did not set out their work rigorously; even those who understood what was required were minimalist in their approach and missed out on both marks.	condone abbreviations
		ii	$\pm\frac{\pi}{6}$ or $-\frac{5\pi}{6}$ soi	M1	may be implied by correct answer or ±0.523598775…, or may appear on quadrant diagram or graph	condone ±30° or − 150°
		ii	$\frac{11\pi}{6}$	A1		ignore extra values outside the range
		ii	$\frac{7\pi}{6}$	A1	if **A0A0**, **SC1** for 1.8333333π and 1.16666666π to 3 or more sf or **SC1** for 330° and 210° www Examiner's Comments Almost half of candidates obtained full marks on this question. Most obtained $\pm\frac{\pi}{6}$ or $\pm 30°$ to earn the first	if full marks or **SC1** awarded, subtract 1 for extra values *in* the range

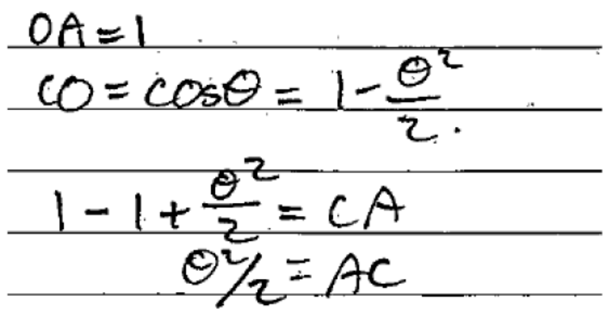
					mark; some obtained the correct angles and left their answers in degrees or only found one of the angles and a few lost a mark by adding extra values, usually $\frac{\pi}{6}$ and / or $\frac{5\pi}{6}$. Over a quarter of candidates failed to score: the usual mistake was a first move of 2θ = $\sin^{-1}(\pm 1)$.	
			Total	5		
3			½ × 12.4^2 × 2.1 (= 161.448)	M1*	or $\pi \times \frac{120.32}{360} \times 12.4^2$	angle in degrees to 3 sf or better
			½ × 12.4^2 × sin2.1 (= 66.3 to 66.4) or ½ × 21.5 (121..) × 6.16 (9…)	M1*	angle in degrees to 3 sf or better	may be implied by 2.81 (7168325…) (degrees) or 2.53 (5559362) (grad)
			their 161.448 – their 66.36	M1dep*		
			95 to 95.1	A1	**Examiner's Comments** This was very well done. By and large the correct formulae were used and the entire solution was worked in radians, nearly always resulting in full marks. Some candidates worked in degrees and then worked with rounded numbers, often following on to over specify their answer and lose the final mark. A significant minority did not use ½r^2½sin½03B8, but used a variety of methods in order to arrive at ½ ×base×height for the area of the triangle. Often this went astray, resulting in a loss of three marks.	if unsupported, B4 for 95.08 (446) r.o.t. to 4 sf or better
			Total	4		
4			1 – $\cos^2 x$ = 3cos$x$ – 2 oe	M1*		
			$\cos^2 x$ + 3cos$x$ – 3 [= 0]	M1*dep	or – cos2 x – 3cos x + 3 = 0	condone one sign error *or* constant term of – 1 (in LH version) or + 1 (in RH version)

			$\cos x =$ their $\dfrac{-3+\sqrt{21}}{2}$ or $\cos x =$ their 0.79 to 0.7913 soi	M1	dependent on award of previous method mark, must be correct for their quadratic	ignore other values (eg −3.79…); $x = 0.791287847\ldots$ but **M0** if no recovery
			[$x =$] 0.6578 to 0.66 isw cao	A1	**A0** for eg 0.66π if 0.66 not seen separately if **A1A1** extra values in range incur a penalty of 1; ignore extra values outside range if **A0A0** allow **SC1** for 37.69 to 37.7° *and* 322 to 322.31° *or* for (0.209 to 0.21)π *and* (1.79 to 1.791)π **Examiner's Comments** A few candidates were unable to eliminate $\sin^2\theta$ legitimately, but all bar the weakest candidates managed at least 2 marks here. A small numberof candidates made errors when rearranging to zero – generally with the constant term.	NB $x = 0.65788395\ldots$ NB $x = 5.625301357\ldots$
			[$x =$] 5.625 to 5.63 isw cao	A1	Some were using x for $\cos x$ in their quadratic formula and not recovering the 'cos'. This was unfortunate. Candidates must realise that this is not a useful practice. Even those who made other substitutions often failed to give their evaluated formula a subject and then confused themselves. Some candidates resorted to rounded decimals very quickly and made premature approximation errors in their answers, thus losing one or more accuracy mark. Quite a few candidates were finding the second angle by adding 0.66 to 1.5π rather than subtracting it from 2π. A fair number of candidates worked in degrees – a good number of these were allowed the SC1 for a pair of correct answers. When the	no **SC** mark available if extra values in range

					question stipulates angles over a range such as "between 0 and 2π", the expectation is that their angleswill be in radians, not degrees.	
			Total	5		
5			27 = ½ r^2 × 1.5 oe	M1	Or $27 = \frac{85.943669...}{360} \times \pi r^2$	angle in degrees rounded to 2 sf or more
			$r = 6$ soi	A1	may be embedded in formula for arc length	may be implied by later work eg 9 or 21
			their r × 1.5	M1	or their $\frac{85.943639}{360} \times 2\pi \times$ their r	if r is incorrect, we must **see** their r × 1.5 [+ 2r] for **M1** if r is correct, **M1** may be implied by 9 or 21
					allow full marks for recovery from working with rounded value of θ in degree form	
			21 [cm] cao	A1	Examiner's Comments This was very well done: approximately two thirds of candidates obtained full marks. Some candidates converted to degrees and lost the accuracy marks and a few candidatesused incorrect formulae.	**B4** for 21 unsupported www
			Total	4		
6		i	$\frac{1}{2}r^2\theta$ or $\frac{1}{2}a^2 \sin\theta$ or $a^2 \sin\frac{1}{2}\theta \cos\frac{1}{2}\theta$ seen	M1	do not allow use of variable other than θ	allow eg
		i	$\frac{1}{2}r^2\theta - \frac{1}{2}a^2 \sin\theta$ isw oe	A1	Examiner's Comments This was generally very well done, but some candidates gave the area of the triangle as ½a^2 and a few gave the area of the sector as $r\theta$.	$\frac{\theta}{2\pi} \times \pi r^2$ or $\frac{1}{2}a^2 \sin\left(\frac{180\theta}{\pi}\right)$ seen oe

		ii	$\frac{1}{2}a^2 \sin 0.8 = \frac{1}{2}\times 12^2 \times 0.8 - \frac{1}{2}a^2 \sin 0.8$ oe [a =] 8.96 cao; mark the final answer	B1 B1	or eg $\frac{1}{2}a^2 \sin 0.8 = \frac{1}{4}\times 12^2 \times 0.8$ [= 28. equivalent in degrees NB θ = 45.8366236…° if unsupported, allow **B2** for 8.96 or allow **B1** for 9.0 or 8.96074…to 4 sf or more **Examiner's Comments** A significant minority were unable to make progress with this part due to incorrect work in part 9(i). Many others set the area of the sector equal to the area of the triangle and failed to score. A few needlessly converted to degrees, and often went wrong either by losing the accuracy mark or making a method error in the formula for the sector. A surprising number of candidates ignored their correct work in part (i) and began again with incorrect expressions.	**NB** $a^2 = \frac{57.6}{0.717356} = 80.29485$ NB θ = 45.83662361…° **NB** $\frac{1}{2}\sin 0.8 = 0.35867..$
			Total	4		
7			Area = $5 = \frac{1}{2}\times 7^2 \times \theta$ θ = $\frac{10}{49}[= 0.204]$	M1(AO3.1a) A1(AO1.1) [2]		
			Total	2		
8		a	Tangent perpendicular to radius	B1(AO2.4) [1]		

		b	Draw horizontal line from B to AD: $\cos A = \frac{6-2}{6+2}$ $\cos A = \frac{1}{2} \Rightarrow A = \frac{1}{3}\pi$	M1(AO3.1a) A1(AO2.1a) [2]	AG Successful completion	
		c	$DC = (6+2)\sin\frac{1}{3}\pi$ $= 4\sqrt{3}$ Trapezium ABCD has area $\frac{1}{2}(6+2)\times 4\sqrt{3}$ $= 16\sqrt{3}\text{ cm}^2$	M1(AO3.1a) A1(AO1.1) M1(AO1.1) A1(AO2.1) [4]	AG	
		d	Angle ABC $= \frac{2}{3}\pi$ $16\sqrt{3} - \frac{1}{2}\times 6^2 \times \frac{1}{3}\pi - \frac{1}{2}\times 2^2 \times \frac{2}{3}\pi$ 4.674 cm^2	M1(AO3.1a) M1(AO1.1) A1(AO1.1) [3]	Either sector correct	

			Total	10	
9			AC = [AO − OC] = 1 − cosθ or cosθ = 1 − *AC* θ small so $\text{AC } 1-\left(1-\frac{\theta^2}{2}\right)=\frac{\theta^2}{2}$	M1(AO 1.1a) E1(AO 2.1) [2]	**AG** Allow AC = AO – OC with *OC* = cosθ for M1 Convincing completion <u>Examiner's Comments</u> This is the first example of a 'Show that' question in this paper and candidates could not score unless they explained the logic of their initial expression Exemplar 1  OA = 1 $CO = \cos\theta = 1 - \frac{\theta^2}{2}$. $1 - 1 + \frac{\theta^2}{2} = CA$ $\theta^2/2 = AC$ The candidate knows the small angle approximation to use but does not explain where their first equation for *CA* comes from. Therefore no marks can be earned.
			Total	2	

10			$\cos C = \dfrac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8} = 0.5$ $C = \frac{1}{3}\pi$ Area triangle $= \frac{1}{2} \times 5 \times 8 \times \sin\frac{1}{3}\pi$ Area of sector $= \frac{1}{2} \times r^2 \times \frac{1}{3}\pi = \frac{1}{2} \times \left(\frac{1}{2} \times 5 \times 8 \sin\frac{1}{3}\right.$ $r^2 = \dfrac{6}{\pi} \times 10 \sin\frac{1}{3}\pi \Rightarrow r = 4.07$ to	M1 (AO 3.1a) A1 (AO 1.1b) B1 (AO 1.1a) M1 (AO 3.1a) A1 (AO 1.1b) [5]	Use of the cosine rule to find angle C Allow 60° May be implied Forming an equation for r^2 Allow 4.1 or better
			Total	5	