1. i. Show that $\cos(\alpha+\beta)=\dfrac{1-\tan\alpha\tan\beta}{\sec\alpha\sec\beta}$.

[3]

ii. Hence show that $\cos 2\alpha=\dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$.

[2]

iii. Hence or otherwise solve the equation $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\dfrac{1}{2}$ for 0° ≤ θ ≤ 180°.

[3]

2. Express 6 cos 2θ + sinθ in terms of sinθ.

Hence solve the equation 6 cos 2θ + sinθ = 0, for 0° ≤ θ ≤ 360°.

[7]

3. In Fig. 5, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all θ.
AB = x and AE = $2x$.

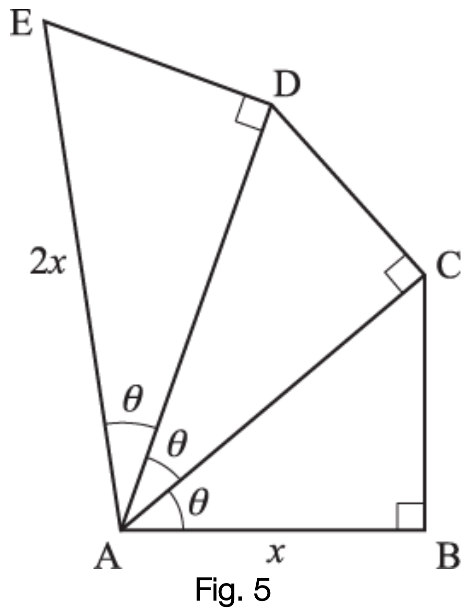

Fig. 5

i. Show that $\sec^3\theta = 2$.

[3]

ii. Hence show the ratio of lengths ED to CB is $2^{\frac{2}{3}} : 1$.

[4]

4. Solve the equation 4 tan θ tan 2θ = 1, for 0° θ < θ < 180°. [4]

5. The day length, Y hours, is defined as the difference between the time the sun rises and the time the sun sets on a particular day. For Manchester, England, the following model is proposed for years which are not leap years.

$$Y = a\sin\left(\frac{2\pi}{365}t + b\right) + c,$$

where t is the time in days since the start of the year and a, b and c are constants.

The maximum value of Y, which is 17.03, occurs on June 21st, when t = 172. The minimum value of Y, which is 7.47, occurs on December 21st, when t = 355.

(a) Show that a = 4.78 and c = 12.25. [2]

(b) Determine the value of b correct to 3 significant figures. [2]

On September 1st, when t = 244, the day length is recorded as 13.76 hours.

(c) Show that the model is a good fit for this value. [2]

In Reykjavik, Iceland, on June 21st the maximum day length was 21.14 hours and on December 21st the minimum day length was 4.12 hours.

[1]

(d) Use this information to refine the model for Manchester to produce a possible model for the day length in Reykjavik.

On September 1st the day length in Reykjavik is recorded as 14.56 hours.

(e) Determine whether your possible model for Reykjavik is a good fit for this value. [1]

6. (See Insert for Jun18 64003.)

(a) In Fig. C1.3, angle CBD = θ. Show that angle CDA is also θ, as given in line 23. [2]

(b) Prove that $h = \sqrt{ab}$, as given in line 24. [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		i	EITHER Use of cos=1/sec (or sin= 1/cosec)	3	Must be **used**
		i	intermediate steFrom RHS	B1	
		i	$\frac{1 - \tan\alpha\ \tan\beta}{\sec\alpha\ \sec\beta}$ $= \frac{1 - \sin\alpha\ /\cos\alpha . \sin\beta\ /\cos\beta}{1/\cos\alpha . 1/\cos\beta}$ $= \cos\alpha\ \cos\beta(1 - \frac{\sin\alpha\ \sin\beta}{\cos\alpha\ \cos\beta})$		
		i		M1	Substituting and simplifying as far as having no fractions within a fraction [need more than $\frac{1 - tt}{\sec\sec} = cc - ss$ ie an intermediate step that can lead to cc-ss]
		i	= cos α cos β – sin α sin β		
		i	= cos(α + β)	A1	Convincing simplification and correct use of cos(α + β) **Answer given**
		i	OR From LHS, cos = 1/sec or sin = 1/cosec **used** $\cos(\alpha + \beta)$ $= \cos\alpha\ \cos\beta - \sin\alpha\ \sin\beta$ $= \frac{1}{\sec\alpha\ \sec\beta} - \sin\alpha\ \sin\beta$	B1	
		i	$= \frac{1 - \sec\alpha\ \sin\alpha\ \sec\beta\ \sin\beta}{\sec\alpha\ \sec\beta}$	M1	Correct angle formula and substitution and simplification to one term OR eg cosαcosβ – sinαsinβ = cos α cos β (1 – tan α tan β)
		i	$= \frac{1 - \tan\alpha\ \tan\beta}{\sec\alpha\ \sec\beta}$	A1	Simplifying to final answer www **Answer given** Or any equivalent work but must have more than cc-ss = answer. **Examiner's Comments** There were some very good solutions here when showing the two

			Answer	Marks	Guidance
					trigonometric expressions were equal. However, the majority were not successful. The most common overall error was not treating both sides of an equation equally. Too often only one side was changed. A common starting point was cos(α+β)=cosαcosβ-sinαsinβ=cosαcosβ-sinαsinβ= 1–tanαtanβ. This was then followed by a confused attempt at dividing by secαsecβ. Candidates need to multiply ‘top and bottom’ by the same thing. Questions that involve ‘Showing’ need more rigour.
		ii	β = α	2	β = α used, Need to see $\sec^2\alpha$ Use of $\sec^2 \alpha = 1 + \tan^2\alpha$ to give required result **Answer Given** Use of $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ soi Simplifying and using $\sec^2\alpha = 1 + \tan^2\alpha$ to final answer **Answer Given** Accept working in reverse to show RHS = LHS, or showing equivalent
		ii	$\cos 2\alpha = \dfrac{1 - \tan^2\alpha}{\sec^2\alpha}$	M1	β = α used, Need to see $\sec^2\alpha$
		ii	$= \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$	A1	Use of $\sec^2 \alpha = 1 + \tan^2\alpha$ to give required result **Answer Given**
		ii	OR, without Hence,		
		ii		M1	Use of $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ soi
		ii	$h = \left(1 - \frac{1}{2}At\right)^2$	A1	Simplifying and using $\sec^2\alpha = 1 + \tan^2\alpha$ to final answer **Answer Given** Accept working in reverse to show RHS = LHS, or showing equivalent **Examiner's Comments** This part was more successful provided that candidates wrote down the identity for $\sec^2\alpha$. There were, however, some long and confused attempts.
		iii	cos 2 θ = ½	M1	Soi or from $\tan^2\theta = 1/3$ oe from $\sin^2\theta$ or $\cos^2\theta$
		iii	i. 2 θ = 60°, 300°, θ = 30°,	A1	First correct solution
		iii	150°	A1	Second correct solution and no others in the range SC B1 for π/6and 5π/6 and no others in the range **Examiner's Comments**

					Most candidates scored the first two marks here. Many failed to give the second solution of 150°.
			Total	8	
2			$\cos 2\theta = 1 - 2\sin^2\theta$	M1*	$\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ (maybe implied in substitution)
			$(6\cos 2\theta + \sin\theta =)\ 6 - 12\sin^2\theta + \sin\theta$	A1	
			$6\cos 2\theta + \sin\theta = 0$ $\Rightarrow 12\sin^2\theta - \sin\theta - 6 = 0$ $\Rightarrow (4\sin\theta - 3)(3\sin\theta + 2) = 0$	M1dep*	Use of correct quadratic equation formula or factorising or comp. the square on their three term quadratic in sinθ (see guidance in question 1 for awarding this method mark) provided $b^2 - 4ac \geq 0$
			$\Rightarrow \sin\theta = 3/4$ or $-2/3$	A1	www
				B1	First correct solution to 1 dp or better (eg 48.59° etc)
				B1	Three correct solutions
			$\Rightarrow \sin\theta = 3/4,\ \theta = 48.6°, 131.4°$ $\sin\theta = -2/3,\ \theta = 221.8°, 318.2°$	B1	All four correct solutions and no others in the range Ignore solutions outside the range **SC** Award max B1B1B0 for answers in radians (0.85, 2.29, 3.87, 5.55 or better – so one correct B1, three correct B1). Award max B1 if there are extra solutions in the range with radians **SC** If M1M1 awarded and both values of $\lvert\sin\theta\rvert \leq 1$ but B0B0B0 then award B1 only for evidence of using $\sin\theta \equiv \sin(180 - \theta)$ **Examiner's Comments** The majority of candidates correctly replaced cos2θ with $1 - 2\sin^2\theta$ although a minority of candidates made the costly mistake of replacing cos2θ with $1 - \sin^2\theta$. While some candidates struggled to factorise $12\sin^2\theta - \sin\theta - 6 = 0$ many used the quadratic formula to solve this equation, and as withquestion 1, there were some candidates who did not state or apply the quadratic formula correctly. While the majority of candidates found the correct values forsinθ some incorrectly obtained $-\frac{3}{4}$ and/or $\frac{2}{3}$. Of those candidates that obtained the correct values for sinθ the majority went on to score full marks. However, it was fairly common to see 'arcsin $\left(-\frac{2}{3}\right)$ = −41.81… *therefore no* *solutions in the range*' with no appreciation that solutions in the correct range could be found from this value. Having found the principal values it was common for candidates to get the other solutions in the range, often sketching the sine curve to help them, though most did this correctly without demonstrating any method.

			Total	7	
3		i	AC = $x \sec \theta$	B1	Accept any equivalent form (e.g. AC cos $\theta = x$). If AC not seen then there must be a diagram as evidence of correct sides - $x\sec\theta$ with no AC is B0
		i	AD = $x \sec^2\theta$ and AE = $x \sec^3\theta$	B1	Accept $2x = x\sec^3\theta$ (as AE = $2x$) or any equivalent form. Otherwise there must be a corresponding diagram as evidence of correct sides. Accept $\cos^3\theta = x / \text{AC} \times \text{AC} / \text{AD} \times \text{AD} / 2x$ for the first two marks
		i	$\Rightarrow x\sec^3\theta = 2x$ $\Rightarrow \sec^3\theta = 2$*	B1	This line (oe) must be seen before the x's cancelled **NB AG** – dependent on all previous marks
		i	**OR** AD = $2x \cos \theta$	B1	Same principles as above for each corresponding mark
		i	AC = $2x\cos^2\theta$ and AB = $2x\cos^3\theta$	B1	or $x = 2x\cos^3\theta$ (as AB = x)
		i	$2x\cos^3\theta = x \Rightarrow \sec^3\theta = 2$*	B1	Must see $2x\cos^3\theta = x$ (oe) before given answer **Examiner's Comments** This question provided a certain amount of discrimination between candidates with some producing clear, concise arguments for why $\sec^3\theta = 2$ and why the ratio of the lengths ED to CB was $2^{\frac{2}{3}}$:1 while a significant number left both parts of this question blank or scored no marks. The majority of candidates, however, scored at least one mark in (i) for starting that AC = $x\sec\theta$ (or equivalent) or that AD = $2x\cos\theta$ but many failed to find corresponding expressions for either AD and AE or AC and AB in terms of x and one of $\sec\theta$ or $\cos\theta$. Examiners noted that many candidates did not make it clear which expression corresponded to which side of the three triangles given in the question making it almost impossible for examiners to award any marks.
		ii	ED = $2x \sin \theta$	B1	oe e.g. $\text{ED} = \sqrt{4x^2 - \text{AD}^2}$ or ED = AD tan θ with AD correctly expressed in terms of x and θ (or using $\theta = 37.5$ or better) - see (i) for alternatives for AD. Allow ED = $1.22x$ (or better) but B0 if ED = … missing
		ii	CB = $x \tan \theta$	B1	oe e.g. $CB = \sqrt{\text{AC}^2 - x^2}$ or CB = AC sin θ with AC correctly expressed in terms of x and θ (or using $\theta = 37.5$ or better) - see (i) for alternatives for AC. Allow CB = $0.77x$ (or better) but B0 if CB = … missing
		ii	$\frac{\text{ED}}{\text{CB}} = \frac{2x\sin\theta}{x\tan\theta} = 2\cos\theta$	B1	www must come from exact working (so not using $\theta = 37.46…$ oe) - accept $\frac{\text{ED}}{\text{CB}} = \frac{2}{\sec\theta}$ or $\frac{\text{ED}}{\text{CB}} = \sec^2\theta$ (oe)

					(as from (i): $\sec^3\theta = 2$)
		ii	$= 2/2^{\frac{1}{3}} = 2^{\frac{2}{3}}$ *	B1	NB AG – dependent on all previous marks in (ii) – must be one step of intermediate working from 2cos θ to given answer **Examiner's Comments** many candidates scored at least two marks for stating that ED = $2x\sin\theta$ and CB = $x\tan\theta$ although many then substituted in the angle from part (i) and tried to derive the exact value of $2^{\frac{2}{3}}$ using approximate values for these two lengths. Candidates who correctly found that $\frac{\text{ED}}{\text{CB}} = 2\cos\theta$ usually went on to obtain the correct ratio although many did not show sufficient steps of working to explain how they obtained the given answer.
			Total	7	
4			$4\tan\theta\tan 2\theta = 1 \Rightarrow 4\tan\theta.\frac{2\tan\theta}{1-\tan^2\theta} = 1$ $\Rightarrow$ 8tanM$^2\theta$ = 1 – $\tan^2\theta$ $\Rightarrow \tan^2\theta = 1/9$ $\tan\theta = 1/3$ or $-1/3$ $\theta = 18.43°$ or $161.57°$ $\theta = 18.43°$ and $161.57°$	M1* M1dep* A1 A1 [4]	Use of double angle formula for tan to get an equation in tan – allow one sign slip only Re-arranges to $\tan^2\theta = k$ where $k > 0$ or attempt to solve $a\tan^2\theta - b = 0$ where $b/a > 0$ One correct answer to at least 1dp Both answers correct to at least 1dp SC A1A0 for answers which round to 0.322 **and** 2.82 (radians) Answers with no working can score B1 B1 (max 2/4) if correct Ignore additional solutions outside the range. If any additional solutions given inside the range of $0 < \theta < 180$ and full marks would have been awarded then remove last mark (so 3/4) **Examiner's Comments** It was pleasing to note that most candidates used the correct double angle formulae for tan 2θ to obtain a correct equation in terms of $\tan\theta$. However, some candidates over complicated the problem by re-writing tan in terms of sin and cos and in these cases it was extremely rare for candidates to make any real significant progress. Of those that correctly re-arranged $4\tan\theta\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = 1$ to $\tan^2\theta = \frac{1}{9}$ it was disappointing that so many candidates then only considered the solutions of the equation $\tan\theta = \frac{1}{3}$ and

					ignored any possible solutions that would have come from the equation $\tan\theta = -\frac{1}{3}$.
			Total	4	
5		a	$a = ½(17.03 − 7.47) = 4.78$ $c + 4.78 = 17.03$ so $c = 12.25$	B1 (AO 3.1b) B1(AO 3.3) [2]	$\sin\theta = 1$ for max $\sin\theta = -1$ for min B1 $17.03 = a + c$, $7.47 = -a + c$ B1 BC Sufficient reasoning needed to justify given answers
		b	$\frac{2\pi}{365} \times 172 + b = \frac{\pi}{2}$ or $\frac{2\pi}{365} \times 355 + b = \frac{3\pi}{2}$ $b = -1.39$	M1 (AO 3.3) A1(AO 1.1) [2]	
		c	$t = 244$ used in their formula $Y = 13.81$ which is fairly close to 13.75 (out by 3.6 minutes)	M1 (AO 3.4) A1(AO 3.5a) [2]	BC
		d	$a = 8.51$ and $c = 12.63$	B1 (AO 3.5b) [1]	
		e	New model gives 15.40 hrs, which is not a good fit	B1 (AO 3.5a) [1]	NB 15.39828…
			Total	8	
6		a	Angle BDC = 90 − θ (angles of triangle) Angle CDA = θ (Angle ADB = 90° as it is the angle in a semicircle)	M1 (AO 2.1) E1 (AO 2.2a) [2]	Including reason Answer given so mark is for reason Reasons can be given in either order Examiner's Comments This question drew on prior knowledge from GCSE. Only few managed to give both 'angles in a triangle' and 'angle in a semicircle'.

		b	Triangle ACD, $\tan\theta = \frac{h}{b}$ Triangle BCD, $\tan\theta = \frac{a}{h}$ $\frac{a}{h} = \frac{h}{b} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$	M1 (AO 1.1) E1 (AO 2.1) [2]	At least one correct expression for tanθ — Alternative method: triangle ACD is similar to triangle DBC Setting expressions equal and correct completion to given answer AG Examiner's Comments This was generally proved correctly.
			Total	4	