

Question	Scheme	Marks	AOs
1(a)	Attempts to use both $\sin(x - 60^\circ) = \pm \sin x \cos 60^\circ \pm \cos x \sin 60^\circ$ $\cos(x - 30^\circ) = \pm \cos x \cos 30^\circ \pm \sin x \sin 30^\circ$	M1	2.1
	Correct equation $2 \sin x \cos 60^\circ - 2 \cos x \sin 60^\circ = \cos x \cos 30^\circ + \sin x \sin 30^\circ$	A1	1.1b
	Either uses $\frac{\sin x}{\cos x} = \tan x$ and attempts to make $\tan x$ the subject E.g. $(2 \cos 60^\circ - \sin 30^\circ) \tan x = \cos 30^\circ + 2 \sin 60^\circ$ Or attempts $\sin 30^\circ$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2 \times \frac{1}{2} - \frac{1}{2}\right) \sin x = \left(2 \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \cos x$	M1	2.1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2} \tan x = \frac{3\sqrt{3}}{2} \Rightarrow \tan x = 3\sqrt{3}$ *	A1*	1.1b
		(4)	
(b)	Deduces that $x = 2\theta + 60^\circ$	B1	2.2a
	$\tan(2\theta + 60^\circ) = 3\sqrt{3} \Rightarrow 2\theta + 60^\circ = 79.1^\circ, 259.1^\circ, \dots$	M1	1.1b
	Correct method to find one value of θ E.g. $\theta = \frac{79.1^\circ - 60^\circ}{2}$	dM1	1.1b
	$\theta = \text{awrt } 9.6^\circ, 99.6^\circ$ (See note)	A1	2.1
		(4)	
(8 marks)			
Notes:			

(a)

M1: Attempts to use both compound angle expansions to set up an equation in $\sin x$ and $\cos x$
The terms must be correct but condone sign errors and a slip on the multiplication of 2

A1: Correct equation $2 \sin x \cos 60^\circ - 2 \cos x \sin 60^\circ = \cos x \cos 30^\circ + \sin x \sin 30^\circ$ o.e.

Note that $\cos 60^\circ = \sin 30^\circ$ and $\cos 30^\circ = \sin 60^\circ$

Also allow this mark for candidates who substitute in their trigonometric values "early"

$$2 \sin x \times \frac{1}{2} - 2 \cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2} \text{ o.e.}$$

M1: Shows the necessary progress towards showing the given result.

There are three key moves, two of which must be shown for this mark.

- uses $\frac{\sin x}{\cos x} = \tan x$ to form an equation in just $\tan x$.
- uses exact numerical values for $\sin 30^\circ, \sin 60^\circ, \cos 30^\circ, \cos 60^\circ$ with at least two correct
- collects terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

A1*: Proceeds to the given answer with accurate work showing all necessary lines.

Examples of two proofs showing all necessary lines

E.g. I $2 \sin x \cos 60^\circ - 2 \cos x \sin 60^\circ = \cos x \cos 30^\circ + \sin x \sin 30^\circ$

$$\sin x (2 \cos 60^\circ - \sin 30^\circ) = \cos x (\cos 30^\circ + 2 \sin 60^\circ)$$

1. collect terms

$$(2 \cos 60^\circ - \sin 30^\circ) \tan x = \cos 30^\circ + 2 \sin 60^\circ$$

2. $\frac{\sin x}{\cos x} = \tan x$ so M1

$$\tan x = \frac{\cos 30^\circ + 2 \sin 60^\circ}{2 \cos 60^\circ - \sin 30^\circ} = \frac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{1}{2}} = 3\sqrt{3}$$

3..uses values and completes proof A1*

E.g II

$$2 \sin x \times \frac{1}{2} - 2 \cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2}$$

1.uses values

$$\Rightarrow \frac{1}{2} \sin x = \frac{3\sqrt{3}}{2} \cos x$$

2.collects terms so M1

$$\Rightarrow \tan x = 3\sqrt{3}$$

3. $\frac{\sin x}{\cos x} = \tan x$ completes proof A1*

(b) Hence

B1: Deduces that $x = 2\theta + 60^\circ$ o.e such as $\theta = \frac{x - 60^\circ}{2}$

This is implied for sight of the equation $\tan(2\theta + 60^\circ) = 3\sqrt{3}$

M1: Proceeds from $\tan(2\theta \pm \alpha^\circ) = 3\sqrt{3} \Rightarrow 2\theta \pm \alpha^\circ = \text{one of } 79.1^\circ, 259.1^\circ, \dots$ where $\alpha \neq 0$

One angle for $\arctan(3\sqrt{3})$ **must** be correct in degrees or radians(3sf). FYI radian answers 1.38, 4.52

dM1: Correct method to find one value of θ from their $2\theta \pm \alpha^\circ = 79.1^\circ$ to $\theta = \frac{79.1^\circ \mp \alpha^\circ}{2}$

This is dependent upon one angle being correct, which must be in degrees, for $\arctan(3\sqrt{3})$

$$\tan(2\theta + 60^\circ) = 3\sqrt{3} \Rightarrow \theta = 9.6^\circ \text{ would imply B1 M1 dM1}$$

A1: $\theta = \text{awrt } 9.6^\circ, 99.6^\circ$ with no other values given in the range

Otherwise: Via the use of $\cos(2\theta + 30^\circ) = \cos 2\theta \cos 30^\circ - \sin 2\theta \sin 30^\circ$.

$$2 \sin 2\theta = \cos(2\theta + 30^\circ) \Rightarrow \tan 2\theta = \frac{\sqrt{3}}{5} \Rightarrow \theta = 9.6^\circ, 99.6^\circ$$

The order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = \frac{\sqrt{3}}{5}$ o.e so allow $\tan 2\theta = \text{awrt } 0.346$ or $\tan 2\theta = \frac{\cos 30^\circ}{2 + \sin 30^\circ}$

Or via double angle identities $\sqrt{3} \tan^2 \theta + 10 \tan \theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities to reach a form $\tan 2\theta = k$ where k is a constant not $3\sqrt{3}$ (or expression in trig terms such as $\cos 30$ etc as seen above)

Or via double angle identities reaches a 3TQ in $\tan \theta$

dM1: Correct order of operations from $\tan 2\theta = k$ leading to $\theta = \dots$

Correctly solves their $\sqrt{3} \tan^2 \theta + 10 \tan \theta - \sqrt{3} = 0$ leading to $\theta = \dots$

A1: $\theta = \text{awrt } 9.6^\circ, 99.6^\circ$ with no other values given in the range.

Note that $\tan(2\theta + 60^\circ) = 3\sqrt{3} \Rightarrow \theta = 9.6^\circ, 99.6^\circ$ is acceptable for full marks