

1. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta \quad (4)$$

(b) Hence solve, for $0 < x < 180^\circ$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

using the identity for $\cos 2\theta$ which will eliminate the 1s

giving your answers to one decimal place where appropriate.

(4)

$$a) \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - (1 - 2\sin^2\theta) + 2\sin\theta\cos\theta}{1 + (2\cos^2\theta - 1) + 2\sin\theta\cos\theta}$$

$$= \frac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta} = \frac{2\sin\theta(\sin\theta + \cos\theta)}{2\cos\theta(\cos\theta + \sin\theta)}$$

$$= \tan\theta \text{ as required}$$

b) using (i), LHS = $\tan 2x$

$$\text{range: } 0 < x < 180^\circ \\ 0 < 2x < 360^\circ$$

$$\tan 2x = 3 \sin 2x \quad \times \cos 2x$$

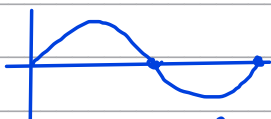
$$\sin 2x = 3 \sin 2x \cos 2x$$

$$0 = 3 \sin 2x \cos 2x - \sin 2x$$

$$0 = \sin 2x (3 \cos 2x - 1) \quad (1)$$

$\therefore$ either $\sin 2x = 0$ or

$$3 \cos 2x - 1 = 0 \quad 2x = \cos^{-1}\left(\frac{1}{3}\right) \\ \Rightarrow \cos 2x = \frac{1}{3} \quad \text{or } 2x = 360^\circ - \cos^{-1}\left(\frac{1}{3}\right)$$



$$2x = 180^\circ, 360^\circ \\ x = 90^\circ \quad (1)$$

out of range

$$x = 35.8^\circ, 144.7^\circ \quad (1)$$

$$x = \{35.8^\circ, 90^\circ, 144.7^\circ\}$$

2. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2 \sin(x - 60^\circ) = \cos(x - 30^\circ)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leq \theta < 180^\circ$

$$2 \sin 2\theta = \cos(2\theta + 30^\circ)$$

giving your answers to one decimal place.

(4)

from formula book:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \end{aligned}$$

$$\begin{aligned} (a) \quad \sin(x - 60^\circ) &= \sin x \cos 60^\circ - \sin 60^\circ \cos x \\ \cos(x - 30^\circ) &= \sin x \sin 30^\circ + \cos x \cos 30^\circ \end{aligned}$$

$$2 \sin x \cos 60^\circ - 2 \sin 60^\circ \cos x = \sin x \sin 30^\circ + \cos x \cos 30^\circ \quad (1)$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2} \quad \cos 60^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$(2 \sin x \times \frac{1}{2}) - (2 \times \frac{\sqrt{3}}{2} \times \cos x) = \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \quad (1)$$

$$\sin x - \sqrt{3} \cos x = \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x$$

collect sin and cos terms

$$\frac{1}{2} \sin x = \left(\frac{\sqrt{3}}{2} + \sqrt{3} \right) \cos x$$

$$\sin x = \sqrt{3} + 2\sqrt{3} \cos x$$

$$\frac{\sin x}{\cos x} = \tan x \quad \left(\begin{array}{l} \sin x = 3\sqrt{3} \cos x \\ \tan x = 3\sqrt{3} \end{array} \right) \quad \frac{\cos x}{\cos x} = 1$$

(b) Using part (a):

$$x - 60 = 2\theta$$

$$x - 30 = 2\theta + 30$$

$$x = 2\theta + 60 \quad (1)$$

$$x = 2\theta + 60$$

So we can use $\tan(x) = 3\sqrt{3}$ with $x = 2\theta + 60$

$$\tan(2\theta + 60) = 3\sqrt{3}$$

$$2\theta + 60 = \tan^{-1}(3\sqrt{3})$$

$$2\theta + 60 = 79.1 \quad (1)$$

$$2\theta = 19.1$$

$$\theta = 9.6 \quad (1) \quad \leftarrow \text{tan repeats every } 90^\circ$$

$$\theta = 9.6^\circ, 99.6^\circ \quad (1)$$

$$\uparrow$$
$$9.6 + 90 = 99.6$$

the next value (189.6) will be outside the
given range of $0 \leq \theta \leq 180$

3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12 \cos \theta \quad 5 + 2 \sin \theta \quad \text{and} \quad 6 \tan \theta$$
$$4 \sin^2 \theta - 52 \sin \theta + 25 = 0$$

(a) show that

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k(1 - \sqrt{3})$$

where k is a constant to be found.

(5)

$$a) \quad r = \frac{5 + 2 \sin \theta}{12 \cos \theta} = \frac{6 \tan \theta}{5 + 2 \sin \theta} \quad (1)$$

$$(5 + 2 \sin \theta)^2 = (6 \tan \theta)(12 \cos \theta)$$

$$(5 + 2 \sin \theta)^2 = 72 \frac{\sin \theta}{\cancel{\cos \theta}} \cancel{\cos \theta} \quad (1)$$

$$25 + 20 \sin \theta + 4 \sin^2 \theta = 72 \sin \theta$$

$$\therefore 4 \sin^2 \theta - 52 \sin \theta + 25 = 0 \quad (1)$$

$$b) (2\sin \theta - 25)(2\sin \theta - 1) = 0$$

$$\sin \theta = \frac{25}{2} \text{ or } \sin \theta = \frac{1}{2} \quad (1)$$

$$\text{since } -1 \leq \sin \theta \leq 1,$$

$$\sin \theta = \frac{1}{2} \text{ only}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore \text{As } \theta \text{ is obtuse, } \theta = \frac{5\pi}{6} \text{ only} \quad (1)$$

$$c) S_{\infty} = \frac{a}{1-r} \text{ for } |r| < 1$$

$$a = 12 \cos \theta = 12 \cos \frac{5\pi}{6} = -6\sqrt{3} \quad (1)$$

$$r = \frac{5 + 2 \sin \left(\frac{5\pi}{6} \right)}{12 \cos \left(\frac{5\pi}{6} \right)} = \frac{-\sqrt{3}}{3} \quad (1)$$

$$\therefore S_{\infty} = \frac{-6\sqrt{3}}{1 - \left(-\frac{\sqrt{3}}{3} \right)} \quad (1)$$

$$= \frac{-18\sqrt{3}}{3 + \sqrt{3}} \times \left(\frac{3 - \sqrt{3}}{3 - \sqrt{3}} \right) \quad (1) \quad \leftarrow \text{to rationalise the denominator}$$

$$= \frac{-54\sqrt{3} + 54}{9 - 3}$$

$$= -9\sqrt{3} + 9$$

$$= 9(1 - \sqrt{3}) \quad (1) \quad \therefore k = 9$$

4. (a) Express $2 \cos \theta + 8 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x$$

$$\cos x + \sin x$$

$$\cos x + 2 \sin x$$

$$x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

$$\begin{aligned} a) \quad 2 \cos \theta + 8 \sin \theta &= R \cos(\theta - \alpha) \\ &= R \cos \theta \cos \alpha + R \sin \theta \sin \alpha \end{aligned}$$

$$R \cos \alpha = 2 \quad \Rightarrow R = \sqrt{2^2 + 8^2} = 2\sqrt{17} \quad (1)$$

$$R \sin \alpha = 8$$

$$\tan \alpha = \frac{8}{2} \quad \Rightarrow \alpha = 1.326 \text{ rad (3dp)} \quad (1)$$

$$\begin{aligned} b) \quad (i) \quad S_9 &= \frac{9}{2} (2a + (9-1)d) & a &= \cos x \\ & & d &= \sin x \\ &= 4.5 (2 \cos x + 8 \sin x) \end{aligned}$$

$$= 4.5 \times 2\sqrt{17} \cos(\theta - 1.326)$$

$$\therefore \max S_9 \text{ is when } \cos(\theta - 1.326) = 1, \text{ and } S_9 = 4.5 \times 2\sqrt{17} \quad (1)$$

$$\max S_9 = 9\sqrt{17} \quad (1)$$

$$(ii) \quad \cos(\theta - 1.326) = 1$$

$$\theta - 1.326 = 0$$

$$\theta = 1.326 \quad (1)$$

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \tan \theta (8 \cos \theta + 23 \sin^2 \theta) = 8 \sin 2\theta (1 + \tan^2 \theta)$$

may be written as

$$\sin 2\theta (A \cos^2 \theta + B \cos \theta + C) = 0$$

where A , B and C are constants to be found.

(3)

(b) Hence, solve for $360^\circ \leq x \leq 540^\circ$

$$2 \tan x (8 \cos x + 23 \sin^2 x) = 8 \sin 2x (1 + \tan^2 x) \quad x \in \mathbb{R} \quad x \neq 450^\circ$$

(4)

$$a) \quad \frac{2 \sin \theta}{\cos \theta} (8 \cos \theta + 23 (1 - \cos^2 \theta)) = 8 \sin 2\theta \times \sec^2 \theta$$

$$2 \sin \theta \cos \theta (8 \cos \theta + 23 - 23 \cos^2 \theta) = 8 \sin 2\theta \quad (1)$$

$$\sin 2\theta (8 \cos \theta + 23 - 23 \cos^2 \theta) = 8 \sin 2\theta \quad (1)$$

$$\sin 2\theta (23 \cos^2 \theta - 8 \cos \theta - 15) = 0 \quad (1)$$

$$b) \quad \sin 2x (23 \cos^2 x - 8 \cos x - 15) = 0$$

$$\sin 2x = 0 \quad \text{or} \quad 23 \cos^2 x - 8 \cos x - 15 = 0$$

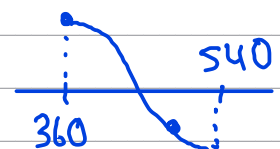
$$\therefore x = 360^\circ$$

$$\text{or } x = 540^\circ \quad (1)$$

$$(\cos x - 1)(23 \cos x + 15) = 0$$

$$\cos x = 1 \Rightarrow x = 360^\circ$$

$$\cos x = -\frac{15}{23} \Rightarrow x = 491^\circ \quad (3\text{sf}) \quad (1)$$



$$x = \{360, 491, 540\} \quad (1)$$