1. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

small angle approximations: $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\sin\theta \approx \theta$

$4\sin\frac{\theta}{2} + 3\cos^2\theta \approx 4\left(\frac{\theta}{2}\right) + 3\left(1 - \frac{\theta^2}{2}\right)^2$ (1)

$= 2\theta + 3\left(1 - \theta^2 + \frac{\theta^4}{4}\right)$

$= 2\theta + 3 - 3\theta^2 + \frac{3}{4}\theta^4$ (1) → ignore θ^4 term because θ is small

$= 3 + 2\theta - 3\theta^2$ (1)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

a) $f'(\alpha) = 0$ ← small angle approximation for $\cos\alpha$

$\therefore 2\alpha + \frac{1}{2}\left(1 - \frac{\alpha^2}{2}\right) = 0$ (1)

$\times 2$ ↓

$4\alpha + 1 - \frac{\alpha^2}{2} = 0$

$\times -2$ ↓

$\alpha^2 - 8\alpha - 2 = 0$ (1)

$\alpha = 8.243$ or $\alpha = -0.243$

choose $\alpha = -0.243$ as α is small. (1)

b) $P(0, 3)$

$f'(0) = \frac{1}{2}\cos 0 = \frac{1}{2}$ ← gradient (m) when $x = 0$.

$y - 3 = \frac{1}{2}(x - 0)$ (1) ← uses $y - y_1 = m(x - x_1)$ with point P.

$y = \frac{1}{2}x + 3$ (1)