1. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

a) $\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta$

$\equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ 1 $\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ 1

$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$ 1 $\equiv \dfrac{2}{\sin 2\theta}$

$\equiv 2\operatorname{cosec}2\theta$

$\Rightarrow \tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta$ as required. 1

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{1}{2}\sin(2\theta) = \frac{\cancel{2}}{\cancel{2}}\sin\theta\cos\theta$

$\operatorname{cosec}2\theta \equiv \dfrac{1}{\sin 2\theta}$

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

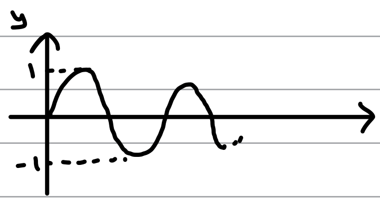
does not have any real solutions.

(1)

$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}(2\theta) \equiv \dfrac{2}{\sin 2\theta} = 1$

$\Rightarrow \sin 2\theta = 2$

$2\theta = \sin^{-1}(2)$



$-1 \leq \sin 2\theta \leq 1$ 1

$2 > 1$

$\Rightarrow$ No Solution exists.

2. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

i) $\sec(x) = \frac{1}{\cos(x)}$

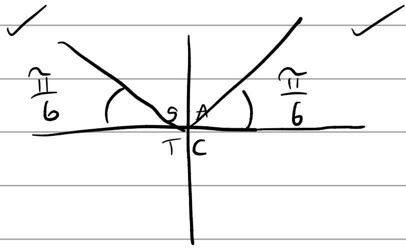
$4\sin(x) = \frac{1}{\cos(x)}$

$4\sin(x)\cos(x) = 1$ $\qquad$ $2\sin(x)\cos(x) = \sin(2x)$

$2\sin(x)\cos(x) = \frac{1}{2}$ $\qquad$ $0 \le x < \frac{\pi}{2}$

$0 \le 2x < \pi$

$\sin(2x) = \frac{1}{2}$



$2x = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}, \quad \frac{5\pi}{6}$

$x = \frac{\pi}{12}, \quad \frac{5\pi}{12}$

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

ii) $\sin(\theta - \alpha) = \sin(\theta)\cos(\alpha) - \sin(\alpha)\cos(\theta)$

$R\sin(\theta - \alpha) = R\sin(\theta)\cos(\alpha) - R\sin(\alpha)\cos(\theta)$

$R\sin(\theta)\cos(\alpha) - R\sin(\alpha)\cos(\theta) = 5\sin(\theta) - 5\cos(\theta)$ ✓1

$\underline{\sin(\theta)}$: $R\cos(\alpha) = 5$ [1]

$\underline{\cos(\theta)}$: $\cancel{-}R\sin(\alpha) = \cancel{-}5$ [2]

$[2] \div [1] \rightarrow \dfrac{\cancel{R}\sin(\alpha)}{\cancel{R}\cos(\alpha)} = \dfrac{5}{5}$

$\tan(\alpha) = 1$

$\alpha = 45°$ ✓1

$[2]^2 + [1]^2 \rightarrow R^2\sin^2(\alpha) + R^2\cos^2(\alpha) = 5^2 + 5^2$

$R^2[\underbrace{\sin^2(\alpha) + \cos^2(\alpha)}_{1}] = 50$

$\sin^2(\alpha) + \cos^2(\alpha) = 1$

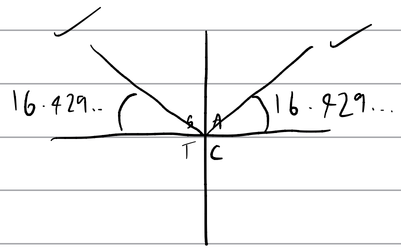
$R^2 = 50$

$R = \sqrt{50} = 5\sqrt{2}$ ✓1

$5\sqrt{2}\sin(\theta - 45) = 2$ $\qquad 0 \leq \theta \leq 360$

$\sin(\theta - 45) = \frac{\sqrt{2}}{5}$

$\theta - 45 = 16.4299\ldots, 163.57\ldots$ ✓1



$\theta = \underline{61.4, 208.6}$ (1 d.p) ✓1

3. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

a) $5\sin 2\theta = 9\tan\theta$

using $\sin 2\theta = 2\sin\theta\cos\theta$

$5(2\sin\theta\cos\theta) = 9\tan\theta$

using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$10\sin\theta\cos\theta = \dfrac{9\sin\theta}{\cos\theta}$ ①

$10\sin\theta\cos^2\theta = 9\sin\theta$

$10\sin\theta\cos^2\theta - 9\sin\theta = 0$

$\sin\theta(10\cos^2\theta - 9) = 0$

$\sin\theta = 0$ OR $10\cos^2\theta - 9 = 0$ ①

$\theta = \sin^{-1}(0)$

$= 0°$

From calculator

$10\cos^2\theta = 9$

$\cos^2\theta = \dfrac{9}{10}$

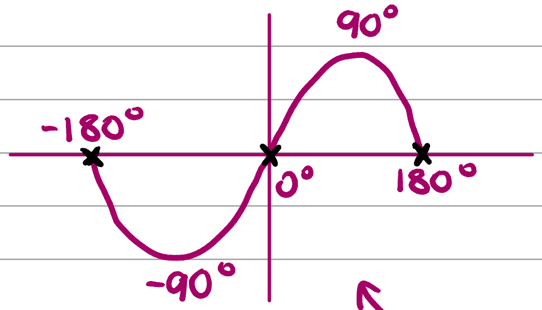
$\cos\theta = \dfrac{\pm 3}{\sqrt{10}}$

From Calculator

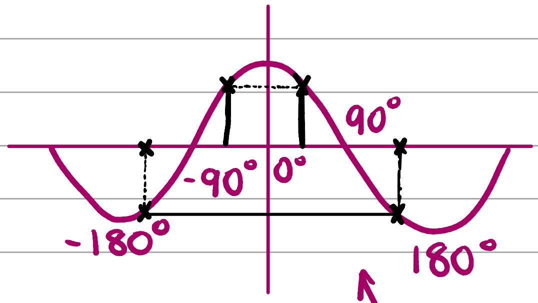
$\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{10}}\right) = 18.4°$

①

$\theta = \cos^{-1}\left(\dfrac{-3}{\sqrt{10}}\right) = 161.6°$



$\sin\theta$



$\cos\theta$

a) Solutions found by using cos θ and sin θ curves

$\theta = 0°, 180°, -180°, 18.4°, -18.4°,$
$161.6°, -161.6°$ 3

b) $\theta = x - 25°$
$x = \theta + 25°$

sub $\theta = -18.4°$

$x = -18.4° + 25°$ 1
$= 6.6°$ 1

4. (a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

(3)

a) $\operatorname{Cosec}\theta - \sin\theta \equiv \frac{1}{\sin\theta} - \sin\theta$

$\equiv \frac{1-\sin^2\theta}{\sin\theta}$ ①

$\equiv \frac{\cos^2\theta}{\sin\theta}$

$\equiv \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\operatorname{Cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$ as required. ①

$\operatorname{Cosec}\theta = \frac{1}{\sin\theta}$ ①

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow 1 - \sin^2\theta = \cos^2\theta$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$

(b) Hence, or otherwise, solve for $0 < x < 180°$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$$

(5)

b) Part a : $\operatorname{Cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$

$\Rightarrow \cos x \cot x = \cos x \cdot \cot(3x - 50°)$

$\div \cos x$ $\qquad \div \cos x$

$\Rightarrow \underbrace{\cot x = \cot(3x-50°)}_{\text{equal}} \Rightarrow x = 3x - 50$ ①

$2x = 50°$

$x = 25°$ ①

Since $\cot x$ has a period of 180°, we can find a second solution

$\Rightarrow x + 180 = 3x - 50$ ①

$\Rightarrow 2x = 230° \Rightarrow x = 115°$ ①

There will be a third solution when $\cos x = 0 \Rightarrow x = \cos^{-1}(0)$

$\Rightarrow x = 90°$ ①

$\Rightarrow x = 25°, \; x = 90°$ and $x = 115°$

5. (a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

$$\cos 3A = \cos(\underset{2A}{2A} + \underset{A}{A})$$

Compound Angle Formula:

- $\cos(x+y) = \cos x \cos y - \sin x \sin y$

We can use the compound angle formula with $x = 2A$ and $y = A$.

Double Angle Formula:

- $\cos 2A = 2\cos^2 A - 1$
- $\sin 2A = 2\sin A \cos A$

$$\Rightarrow \cos 3A = \cos 2A \cos A - \sin 2A \sin A \quad ①$$
$$= (2\cos^2 A - 1)\cos A - (2\sin A\cos A)\sin A \quad ①$$
$$= 2\cos^3 A - \cos A - 2\sin^2 A\cos A$$
$$= 2\cos^3 A - \cos A - (2 - 2\cos^2 A)\cos A \quad ①$$
$$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$$

- $\sin^2 x + \cos^2 x = 1.$

$\Rightarrow 2\sin^2 x = 2 - 2\cos^2 x$

$$\Rightarrow \cos 3A \equiv 4\cos^3 A - 3\cos A \quad ① \text{ as required.}$$

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

$$1 - \cos 3x = \sin^2 x$$

- $\sin^2 x + \cos^2 x = 1$
- Part a: $\cos 3A \equiv 4\cos^3 A - 3\cos A$

$$\Rightarrow 1 - \cos 3x = 1 - \cos^2 x$$

let $\cos x = y$

$\Rightarrow -4y + y + 3$

$-(4y - y - 3)$

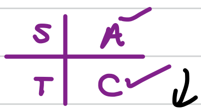
$-(4y+3)(y-1)$

$$\Rightarrow 1 - (4\cos^3 x - 3\cos x) = 1 - \cos^2 x$$

$$\Rightarrow 1 - 4\cos^3 x + 3\cos x - 1 + \cos^2 x = 0 \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0 \quad ①$$

$$\Rightarrow \cos x(\cos x + 3 - 4\cos^2 x) = 0$$

$$\Rightarrow \cos x(4\cos x + 3)(\cos x - 1) = 0$$



$\Rightarrow \cos x = 0$, $x = 90°$, $x = -90$ ①

$4\cos x + 3 = 0$, $\cos x = -\frac{3}{4} \Rightarrow x = 139°$

$\cos x - 1 = 0$ ①, $x = 0$

$\Rightarrow$ Solutions are: $x = -90°, 0°, 90°$ and $139°$. ①

6. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.
(3)

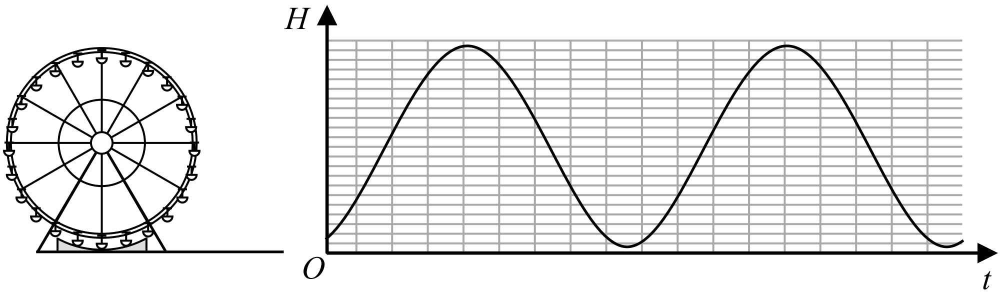


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.
(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?
(1)

a) $R\cos(\theta+\alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$10\cos\theta - 3\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$10\cos\theta = R\cos\theta\cos\alpha$ $\qquad$ $-3\sin\theta = -R\sin\theta\sin\alpha$

$R\cos\alpha = 10$ – ① $\qquad$ $R\sin\alpha = 3$ – ②

$R^2\cos^2\alpha = 100$ $\qquad$ $R^2\sin^2\alpha = 9$

$R^2\cos^2\alpha + R^2\sin^2\alpha = 100 + 9$ $\qquad$ $\cos^2\alpha + \sin^2\alpha \equiv 1$

$R^2(\cos^2\alpha + \sin^2\alpha) = 109$

$R^2 = 109$

a) $R = \sqrt{109}$ – 1

2 ÷ 1

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \frac{3}{10}$

$\tan\alpha = \frac{3}{10}$ – 1

$\alpha = 16.70°$ (2d.p.)

– 1

$= \sqrt{109}\cos(\theta + 16.70°)$

b) i) $1 = a - 10\cos(0) + 3\sin(0)$

$1 = a - 10$

$a = 11$

$H = 11 - 10\cos(80t)° + 3\sin(80t)°$

$10\cos\theta - 3\sin\theta$

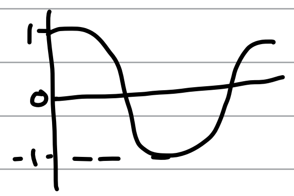
$= \sqrt{109}\cos(\theta + 16.70°)$

$-10\cos(80t)° + 3\sin(80t)°$

$= -(10\cos(80t)° - 3\sin(80t)°)$

$= -\sqrt{109}\cos(80t + 16.70)°$

$H = 11 - \sqrt{109}\cos(80t + 16.70)°$ – 1



ii) $\cos(80t + 16.70)° = -1$

$H = 11 - (\sqrt{109} \times -1)$

$H = 11 + \sqrt{109}$ – 1

c) $\cos(80t + 16.70)° = -1$

$80t + 16.70 = 180$ for first cycle

$180 + 360 = 540$ for second cycle

$80t + 16.70 = 540$ – 1

$t = \frac{540 - 16.7}{80}$ – 1

0.54×60

$= 32$ (2s.f)

c) $t = 6.54$ minutes

$t = 6$ minutes 32 seconds - 1

d) $H = 11 - \sqrt{109}\cos(80t + 16.70)^\circ$

We would increase the '80t' value - for example to 90t. - 1

7. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

a) 6:30 am $\rightarrow$ 6h 30min or 6.5h

$5 + 2\sin(30 \times 6.5)$

$= 4.48$ m ✓

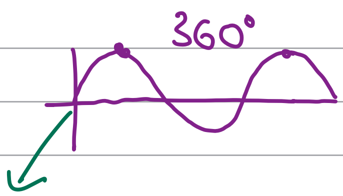
b) $D = 5 + 2\sin(30t)$ find at what t is $D = 3.8$m

$3.8 = 5 + 2\sin(30t)$

$-1.2 = 2\sin(30t)$

$-0.6 = \sin(30t)$

$\sin(30t) = -0.6$ ✓



$30t = \sin^{-1}(-0.6) = -36.869\ldots$ $t > 0$

$30t = -36.86\ldots + 360$ ✓ $\therefore 30t > 0$

$= 323.13\ldots$

$t = \dfrac{323.13\ldots}{30} = 10.77$h ✓ $\rightarrow$ 10h 0.77×60 min

$= 10$h 46min

the boat can leave at 10:46 am ✓

8. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

a) $RHS = \tan\theta \sin 2\theta \equiv \frac{\sin\theta}{\cancel{\cos\theta}}(2\sin\theta\cancel{\cos\theta})$

$\equiv 2\sin^2\theta$

but $\cos 2\theta \equiv 1 - 2\sin^2\theta \therefore 2\sin^2\theta \equiv 1 - \cos 2\theta$ //

hence $RHS \equiv 2\sin^2\theta \equiv 1 - \cos 2\theta$
$= LHS$

b) $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$

$(\sec^2 x - 5)(\cancel{\tan x \sin 2x}) = 3\tan x(\cancel{\tan x \sin 2x})$

$\sec^2 x - 5 = 3\tan x$

$\sec^2 x \equiv 1 + \tan^2 x$

$1 + \tan^2 x - 5 = 3\tan x$

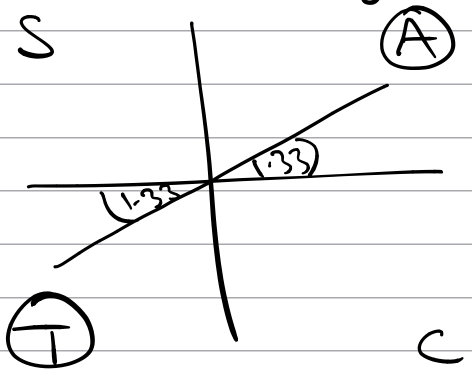
$\tan^2 x - 3\tan x - 4 = 0$

$(\tan x - 4)(\tan x + 1) = 0$

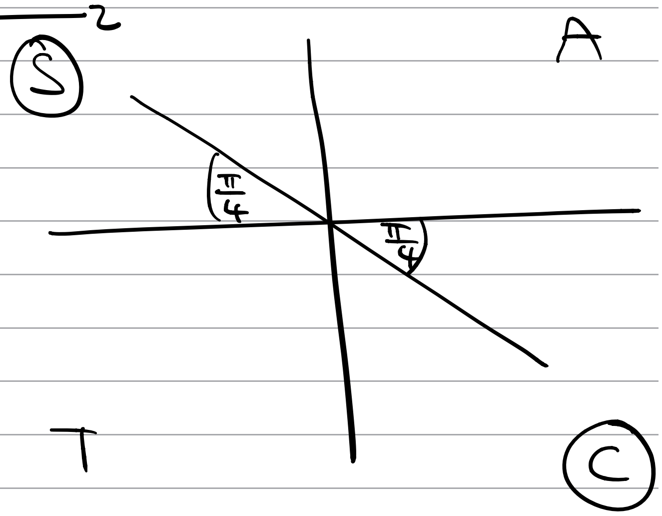
$\tan x = 4$, $\tan x = -1$

$x = \tan^{-1}(4) = 1.326$, $x = \tan^{-1}(-1) = -\frac{\pi}{4}$

range: $-\frac{\pi}{2} < x < \frac{\pi}{2}$



$\boxed{x = 1.326}$



$\boxed{x = -\frac{\pi}{4}}$

9. (a) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to one decimal place.

(3)

a) $\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$

Trig Identities :

• $\cos(a-b) = \cos a\cos b + \sin a \sin b$
$\hookrightarrow a = 3\theta,\ b = \theta$

$\Rightarrow \frac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ 1

$\Rightarrow \frac{\cos(3\theta-\theta)}{\sin\theta\cos\theta} = \frac{\cos(2\theta)}{\sin\theta\cos\theta}$ 1

• $\sin 2\theta = 2\sin\theta\cos\theta$
$\Rightarrow \frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

• $\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} = \cot\theta$

1

$= \frac{\cos(2\theta)}{\frac{1}{2}\sin(2\theta)} = \frac{2}{\tan 2\theta}$

$= 2\cot 2\theta$

$\Rightarrow \frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$ as required. 1

b) $\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$

Part a : $\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 2\cot 2\theta = \frac{2}{\tan 2\theta}$

1

$\Rightarrow 2\cot 2\theta = 4 \Rightarrow \frac{2}{\tan 2\theta} = 4 \Rightarrow \tan 2\theta = \frac{1}{2}$

S | A
T | C

$\Rightarrow 2\theta = \tan^{-1}\left(\frac{1}{2}\right)$ 1

$180 + 26.6$

$\Rightarrow 2\theta = 26.6°$ and $2\theta = 206.6°$

$\Rightarrow \theta = 13.3°$ and $\theta = 103.3°$

$\Rightarrow 13.3°$ is not a valid solution since out of range $\Rightarrow \theta = 103.3°$ 1

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

10. (a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

(a) $\dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$

1 1

$\dfrac{1 - (1 - 2\sin^2\theta) + 2\sin\theta\cos\theta}{1 + (2\cos^2\theta - 1) + 2\sin\theta\cos\theta} \equiv \tan\theta$

$\dfrac{1 - 1 + 2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta} = \tan\theta$

1

$\dfrac{\cancel{2}\sin\theta\,(\cancel{\sin\theta + \cos\theta})}{\cancel{2}\cos\theta\,(\cancel{\cos\theta + \sin\theta})} \equiv \tan\theta$

1

$\dfrac{\sin\theta}{\cos\theta} \equiv \tan\theta$

$\tan\theta \equiv \tan\theta$ ※

DO NOT WRITE IN THIS AREA

(b) Let $\theta = 2x$

$\tan 2x = 3 \sin 2x$ (1)

$\frac{\sin 2x}{\cos 2x} = 3 \sin 2x$

$\sin 2x = 3 \sin 2x \cos 2x$

$\sin 2x - 3 \sin 2x \cos 2x = 0$

$0^\circ < x < 180^\circ$
$0^\circ < 2x < 360^\circ$

$\sin 2x \, (1 - 3 \cos 2x) = 0$

$\sin 2x = 0$ $\quad$ $1 - 3\cos 2x = 0$ (1)

$\cos 2x = \frac{1}{3}$

$\sin 2x = 0$: $2x = 180^\circ$ we don't include 0 because of the range $0 < 2x < 360$

$\therefore \; x = 90^\circ$

$\cos 2x = 1/3$: $2x = 70.52\ldots, \; 289.47\ldots$

$x = 35.26\ldots, \; 144.736\ldots$

$x \approx 35.3^\circ, \; 144.7^\circ$

$\therefore \; x = 35.3^\circ, 90^\circ, 144.7^\circ$ (1dp) (1)

(1)

11. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

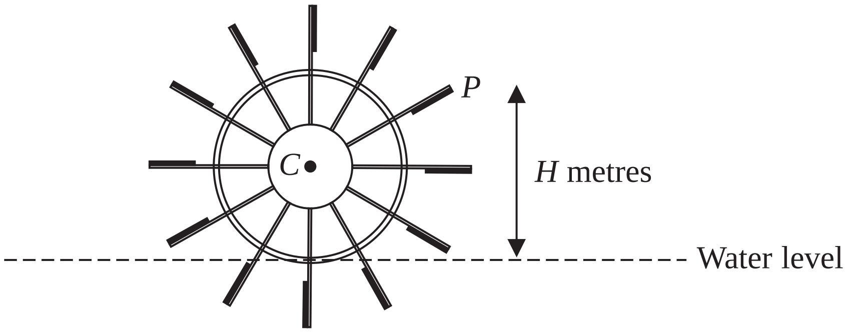


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

Question continued

a) $2\cos\theta - \sin\theta = R\cos(\theta + \alpha)$

$a\cos\theta \pm b\sin\theta$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

convert using formula:
$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$

$R = \sqrt{a^2 + b^2}$

$= \sqrt{2^2 + (-1)^2}$

$= \sqrt{5}$ (1)

to find α, compare like terms:

$\cos\theta$: $2 = R\cos\alpha$

$\cos\alpha = 2/R$

$\sin\theta$: $1 = R\sin\alpha$

$\sin\alpha = 1/R$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1/R}{2/R}$

$= \dfrac{1}{2}$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 0.464$ (3 d.p)

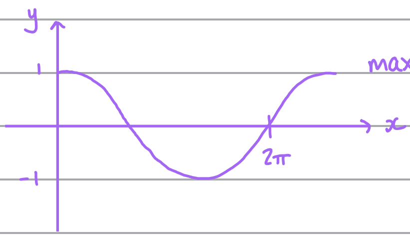
[must be within given range]

b) i) $H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$

$H = 3 + 2[2\cos(0.5t) - \sin(0.5t)]$

$H = 3 + 2\sqrt{5}\cos(0.5t + 0.464)$

using answer from a)



max $\cos x = 1$ so max $H = 3 - 2\sqrt{5}(1)$

$H_{max} = 3 + 2\sqrt{5}$ (1)

Question continued

ii) $\cos(0.5t + 0.464) = 1$ ← using max value from graph

$0.5t + 0.464 = \cos^{-1}(1) = 2\pi$ (1)

$0.5t = 2\pi - 0.464$

$t = 2(2\pi - 0.464)$

$t = 11.6\,s$ (1)

c) $3 + 2\sqrt{5}\cos(0.5t + 0.464) = 0$

$\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}}$ (1)

$0.5t + 0.464 = \cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$

$t = 2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right) - 0.464\right)$ (1)

So the time required is.

(1)

$2(3.977 - 0.464) - 2(2.306 - 0.464) = 3.34$ (1)

d) The '3' would need to vary. (1)

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