1. (a) Use integration by parts to find

$$\int x \cos 2x \, dx .$$

(4)

(b) Hence, or otherwise, find

$$\int x \cos^2 x \, dx .$$

(3)
(Total 7 marks)

2. (a) Use the formulae for sin ($A \pm B$), with $A = 3x$ and $B = x$, to show that 2 sin x cos $3x$ can be written as sin px – sin qx, where p and q are positive integers.

(3)

(b) Hence, or otherwise, find $\int 2 \sin x \cos 3x \, dx$.

(2)

(c) Hence find the exact value of $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2 \sin x \cos 3x \, dx$

(2)
(Total 7 marks)

3. (a) Use the identity for cos ($A + B$) to prove that cos $2A = 2\cos^2 A - 1$.

(2)

(b) Use the substitution $x = 2\sqrt{2}\sin\theta$ to prove that

$$\int_2^{\sqrt{6}} \sqrt{(8 - x^2)}\ \mathrm{d}x = \tfrac{1}{3}(\pi + 3\sqrt{3} - 6).$$

(7)

A curve is given by the parametric equations

$$x = \sec\theta, \quad y = \ln(1 + \cos 2\theta), \quad 0 \le \theta < \frac{\pi}{2}.$$

(c) Find an equation of the tangent to the curve at the point where $\theta = \frac{\pi}{3}$.

(5)
(Total 14 marks)

4. On separate diagrams, sketch the curves with equations

(a) $y = \arcsin x, \quad -1 \le x \le 1$,

(b) $y = \sec x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{3}$, stating the coordinates of the end points of your curves in each case.

(4)

Use the trapezium rule with five equally spaced ordinates to estimate the area of the region bounded by the curve with equation $y = \sec x$, the x-axis and the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, giving your answer to two decimal places.

(4)
(Total 8 marks)

1. (a) Attempt at integration by parts, i.e. $kx \sin 2x \pm \int k \sin 2x dx$, with $k = 2$ or ½

$= \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$ A1

Integrates $\sin 2x$ correctly, to obtain $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$ A1 4

(penalise lack of constant of integration first time only)

(b) Hence method: Uses $\cos 2x = 2\cos^2 x - 1$ to connect integrals B1

Obtains

$\int x \cos^2 x dx = \frac{1}{2}\{\frac{x^2}{2} + \text{answer to part}(a)\} = \frac{x^2}{4} + \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + k$ M1A1 3

Otherwise method

$\int x \cos^2 x dx = x\left(\frac{1}{4} \sin 2x + \frac{x}{2}\right) - \int \frac{1}{4} \sin 2x + \frac{x}{2} dx$ B1,

B1 for $(\frac{1}{4}\sin 2x + \frac{x}{2})$

$= \frac{x^2}{4} + \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + k$ A1 3

[10]

2. (a) $\sin(3x + x) = \sin 3x \cos x + \cos 3x \sin x$

$\sin(3x - x) = \sin 3x \cos x - \cos 3x \sin x$ A1

(subtract) $\Rightarrow$ $\underline{\sin 4x - \sin 2a = 2\sin x \cos 3x}$ A1c.s.o. 3

(b) $\int 2\sin x \cos 3x \, dx = \int(\sin 4x - \sin 2x)dx$

$= \underline{-\frac{\cos 4x}{4} + \frac{\cos 2x}{2} + c}$ A1ft 2

ft their p, q

(c) $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2 \sin x \cos 3x dx = \left(-\frac{1}{4} \cos \frac{10\pi}{3} + \frac{1}{2} \cos \frac{5\pi}{3}\right) - \left(-\frac{1}{4} \cos 2\pi + \frac{1}{2} \cos \pi\right)$

$= \underline{\frac{9}{8}}$ A1 2

[7]

3. (a) $\cos(A + A) = \cos^2 A - \sin^2 A$

$= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$ A1 2

(b) $[x = 2,\ \theta = \frac{\pi}{4};\ x = \sqrt{6},\ \theta = \frac{\pi}{3}]$ B1

$x = 2\sqrt{2}\sin\theta,\ \frac{dx}{d\theta} = 2\sqrt{2}\cos\theta$ B1

$\int\sqrt{8 - x^2}\,dx = \int 2\sqrt{2}\cos\theta\, 2\sqrt{2}\cos\theta\, d\theta = \int 8\cos^2\theta\, d\theta$ A1

Using $\cos 2\theta = 2\cos^2\theta - 1$ to give $\int 4(1 + \cos 2\theta)\,d\theta$ dM1

$= 4\theta + 2\sin 2\theta$ A1 ft

Substituting limits to give $\frac{1}{3}\pi + \sqrt{3} - 2$ or given result A1 7

(c) $\frac{dy}{d\theta} = \frac{-2\sin 2\theta}{1 + \cos 2\theta}$ B1

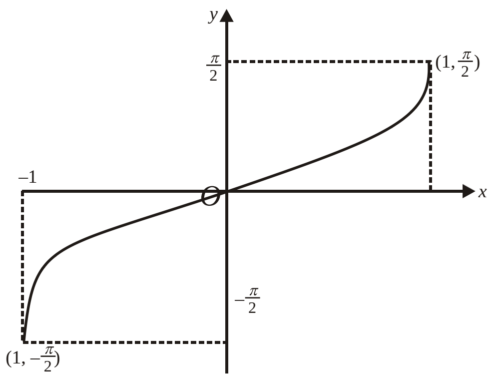
Using the chain rule, with $\frac{dy}{d\theta} = \sec\theta\tan\theta$ to give $\frac{dy}{dx}$ $(= -2\cos\theta)$

Gradient at the point where $\theta = \frac{\pi}{3}$ is –1. A1 ft

Equation of tangent is $y + \ln 2 = -(x - 2)$ (o.a.e.) A1 5

[12]

4. (a)



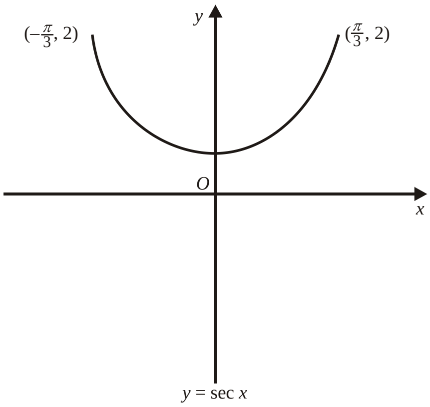
$y = \arcsin x$

(a) Shape correct

passing through O: G1;

end-points: G1 2

(b)



Shape correct,
symmetry in *Oy*: G1
end-points: G1 2

(c)

x	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$\sec x$	2	1.155	1	1.155	2

Area estimate = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x \, dx = \frac{\pi}{6}\left[\frac{2+2}{2} + 1.155 + 1 + 1.155\right]$ A1 A1

= 2.78 (2 d.p.) A1 4

[8]

1. (a) This was a straightforward integration by parts, which was recognised as such and done well in general. The most common error was the omission of the constant of integration, but some confused signs and others ignored the factors of two.

(b) This was done well by those students who recognised that $\cos^2 x = (1 + \cos 2x)/2$ but there was a surprisingly high proportion who were unable to begin this part. Lack of care with brackets often led to errors so full marks were rare. There was also a large proportion of candidates who preferred to do the integration by parts again rather than using their answer to (a).

2. Candidates commonly misunderstood that both the formulae for $\sin(A+B)$ and $\sin(A–B)$ were required by the rubric. The vast majority chose only one, prohibiting progress and usually abandoned the question at this stage. Many who did not complete part (a) attempted to integrate by parts, usually twice, in part (b) before leaving unfinished working. The more successful, or those with initiative, continued with both parts (b) and (c), either with their *p* and *q* values, the letters *p* and *q*, or hopefully guessed *p* and *q* values.

3. Most candidates understood the requirements of the proof of the double angle formula in part (a). Part (b) proved to be discriminating, but a large number of candidates produced good solutions, where they changed the variables and the limits and used the appropriate double angle formula to perform the integration. Some difficulties were experienced differentiating the log function in part (c), but again there were a large number of correct solutions. A few candidates eliminated the parameter and found the cartesian equation of the curve before differentiation.

4. No Report available for this question.