TRIGONOMETRY

1 **a** Given that $4 \sin x + \cos x = 0$, show that $\tan x = -\frac{1}{4}$.

b Hence, find the values of x in the interval $0 \leq x \leq 360°$ for which

$$4 \sin x + \cos x = 0,$$

giving your answers to 1 decimal place.

2 **a** Show that

$$5 \sin^2 x + 5 \sin x + 4 \cos^2 x \equiv \sin^2 x + 5 \sin x + 4.$$

b Hence, find the values of x in the interval $0 \leq x \leq 360°$ for which

$$5 \sin^2 x + 5 \sin x + 4 \cos^2 x = 0$$

3 Solve each equation for x in the interval $0 \leq x \leq 360°$.

Give your answers to 1 decimal place where appropriate.

a $2 \sin x - \cos x = 0$

b $3 \sin x = 4 \cos x$

c $\cos^2 x + 3 \sin x - 3 = 0$

d $3 \cos^2 x - \sin^2 x = 2$

e $2 \sin^2 x + 3 \cos x = 3$

f $3 \cos^2 x = 5(1 - \sin x)$

g $3 \sin x \tan x = 8$

h $\cos x = 3 \tan x$

i $3 \sin^2 x - 5 \cos x + 2 \cos^2 x = 0$

j $2 \sin^2 x + 7 \sin x - 2 \cos^2 x = 0$

k $3 \sin x - 2 \tan x = 0$

l $\sin^2 x - 9 \cos x - \cos^2 x = 5$

4 Solve each equation for θ in the interval $-\pi \leq \theta \leq \pi$ giving your answers in terms of π.

a $4 \cos^2 \theta = 1$

b $4 \sin^2 \theta + 4 \sin \theta + 1 = 0$

c $\cos^2 \theta + 2 \cos \theta - 3 = 0$

d $3 \sin^2 \theta - \cos^2 \theta = 0$

e $4 \sin^2 \theta - 5 \sin \theta + 2 \cos^2 \theta = 0$

f $\sin^2 \theta - 3 \cos \theta - \cos^2 \theta = 2$

5 Prove that

a $(\sin x + \cos x)^2 \equiv 1 + 2 \sin x \cos x$

b $\frac{1}{\cos x} - \cos x \equiv \sin x \tan x, \quad \cos x \neq 0$

c $\frac{\cos^2 x}{1 - \sin x} \equiv 1 + \sin x, \quad \sin x \neq 1$

d $\frac{1 + \sin x}{\cos x} \equiv \frac{\cos x}{1 - \sin x}, \quad \cos x \neq 0$

6 **a** Prove the identity

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 \equiv 2 + \tan^2 x.$$

b Hence find, in terms of π, the values of x in the interval $0 \leq x \leq 2\pi$ such that

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 = 3.$$

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$$\text{f}(x) \equiv \cos^2 x + 2 \sin x, \quad 0 \leq x \leq 2\pi.$$

a Prove that f(x) can be expressed in the form

$$\text{f}(x) = 2 - (\sin x - 1)^2.$$

b Hence deduce the maximum value of f(x) and the value of x for which this occurs.

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