

1.

(i) Show that the equation $\frac{\tan \theta}{\cos \theta} = 1$ may be rewritten as $\sin \theta = 1 - \sin^2 \theta$

[2]

(ii) Hence solve the equation $\frac{\tan \theta}{\cos \theta} = 1$ for $0^\circ \leq \theta < 360^\circ$.

[3]

2.

Simplify $\frac{\sqrt{1 - \cos^2 \theta}}{\tan \theta}$, where θ is an acute angle.

[3]

3.

(i) Show that, when x is an acute angle, $\tan x \sqrt{1 - \sin^2 x} = \sin x$.

[2]

(ii) Solve $4 \sin^2 y = \sin y$ for $0^\circ \leq y < 360^\circ$.

[3]

END OF QUESTION PAPER

Question		Answer/Indicative content	Marks	Part marks and guidance	
1	i	$\frac{\sin \theta}{\cos \theta} = 1$ oe $\frac{\sin \theta}{\cos \theta}$	M1		
	i	$\sin \theta = \cos^2 \theta$ and completion to given result	A1	www Examiner's Comments Many candidates answered this question well, although there were a number of attempted fudges using $\tan \theta = \frac{\cos \theta}{\sin \theta}$. Some adopted a scattergun approach and it was not always possible to follow their method.	
	ii	$\sin^2 \theta + \sin \theta - 1 [= 0]$	M1	allow 1 on RHS if attempt to complete square	condone $y^2 + y - 1 = 0$
	ii	$[\sin \theta =] \frac{-1 \pm \sqrt{5}}{2}$ oe may be implied by correct answer	A1	may be implied by correct answers	mark to benefit of candidate ignore any work with negative root & condone omission of negative root with no comment eg M1 for 0.618...

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	[$\theta =$] 38.17..., or 38.2 and 141.83..., 141.8 or 142	A1	<p>Ignore extra values outside range, A0 if extra values in range or in radians</p> <p>NB 0.6662 and 2.4754 if working in radian mode earns M1A1A0</p> <p>Examiner's Comments</p> <p>This defeated a significant minority of candidates. However, many obtained the correct quadratic equation. Most then went on to attempt factorisation, going wrong and failing to score. A minority successfully completed the square or used the formula. Many of these went on to score full marks, but some candidates missed the last mark because they presented extra values in the range, or because they didn't realise that further work was needed after obtaining the roots of the quadratic.</p>	<p>If unsupported, B1 for one of these, B2 for both. If both values correct with extra values in range, then B1.</p> <p>NB 0.6662 and 2.4754 to 3sf or more</p>
		Total	5		

Question	Answer/Indicative content	Marks	Part marks and guidance
2	$\frac{\sqrt{\sin^2 \theta}}{\sin \theta} \text{ or } \frac{\cos \theta \sqrt{\sin^2 \theta}}{\sin \theta}$ $\frac{\cos \theta}{\cos \theta}$	M1 M1 A1	<p>correct substitution for numerator</p> <p>allow maximum of M1M1 if $\pm\sqrt{\sin^2 \theta}$ oe substituted</p> <p>correct substitution for denominator</p> <p>A0 if follows wrong working or B3 www or if unsupported</p> <p>Examiner's Comments</p> <p>Most candidates recognised one of the trigonometric identities required, and then made no further progress. Of those who spotted both relationships, a good proportion made a mess of simplifying the fraction, often resulting in a final answer of $\frac{1}{\cos \theta}$. A surprising number tried squaring top and bottom, or concocted an equation which they attempted to solve.</p> <p>mark the final answer but ignore attempts to solve for θ allow recovery from omission of θ</p>
	Total	3	

Question		Answer/Indicative content	Marks	Part marks and guidance	
3	i	substitution of $\tan x = \frac{\sin x}{\cos x}$ or $\sqrt{1 - \sin^2 x} = \sqrt{\cos^2 x}$ or $\cos x$ in given LHS	M1	if no substitution, statements must follow a logical order and the argument must be clear; if one substitution made correctly, condone error in other part of LHS	condone omission of variable throughout for M1 only, but allow recovery from omission of variable at end
	i	both substitutions seen and completion to $\sin x$ as final answer	A1	NB AG ; answer must be stated allow consistent use of other variable eg θ for both marks	M0 if first move is to square one or both sides Simply stating eg $\tan x = \frac{\sin x}{\cos x}$ is insufficient
	i			Examiner's Comments A significant minority of candidates chose to work backwards, but few were successful. Many candidates "started at both ends" and tried to meet in the middle – sometimes a method mark was achieved. A good number of candidates earned the first method mark with one of the correct substitutions, but either failed to complete the argument or tried to show something else.	<i>Alternatively SC2</i> for complete argument eg $\tan x = \frac{\sin x}{\cos x}$ [$\tan x \times \cos x = \sin x$] $\sin^2 x + \cos^2 x = 1$ $\cos x = \sqrt{1 - \sin^2 x}$ $\tan x = \frac{\sin x}{\sqrt{1 - \sin^2 x}}$ $\tan x \times \sqrt{1 - \sin^2 x} = \sin x$ oe
	ii	0, 180, 360	B1	all 3 required	NB $\sin y = 0$ or $\frac{1}{4}$
	ii	14 or 14.47 to 14.5	B1	radians: mark as scheme but deduct one from total	ignore extra values outside range
	ii	166 or awrt 165.5	B1	0, π , 2π ; 0.25 or 0.253 or awrt 0.2527; 2.89 or 2.889 or awrt 2.8889 Examiner's Comments Most candidates solved the quadratic successfully and went on to find 14.5 and 166. A surprising number omitted one or more of the three other roots, however.	if B3 , deduct 1 mark for extra values within range

Question			Answer/Indicative content	Marks	Part marks and guidance
			Total	5	