

1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos \theta} + \tan \theta \equiv \frac{\cos \theta}{1 - \sin \theta} \quad \theta \neq (2n + 1)90^\circ \quad n \in \mathbb{Z} \quad (3)$$

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90^\circ$

$$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$$

giving your answers to one decimal place.

(5)

LHS :

$$\frac{1}{\cos \theta} + \tan \theta$$

$$= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \quad \swarrow \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1 + \sin \theta}{\cos \theta} \left(\frac{1 - \sin \theta}{1 - \sin \theta} \right) \quad \swarrow \begin{array}{l} \text{method to get} \\ (1 - \sin^2 \theta) \text{ at numerator} \end{array}$$

$$= \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$= \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)} \quad \swarrow 1 - \sin^2 \theta = \cos^2 \theta$$

$$= \frac{\cos \theta}{1 - \sin \theta} = \text{RHS}$$

$$b) \frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x \quad \text{where } 0^\circ < x < 90^\circ$$

$$\text{for } 2x: 0^\circ < 2x < 180^\circ$$

$$\text{from a) } \frac{1}{\cos \theta} + \tan \theta = \frac{\cos \theta}{1 - \sin \theta}$$

$$\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x \quad (1)$$

$$\cos 2x = 3 \cos 2x (1 - \sin 2x) \quad (1)$$

$$\cos 2x - 3 \cos 2x (1 - \sin 2x) = 0$$

$$\cos 2x (1 - 3(1 - \sin 2x)) = 0$$

$$\cos 2x (3 \sin 2x - 2) = 0$$

$$\cos 2x \neq 0, \quad 3 \sin 2x - 2 = 0$$

$$\sin 2x = \frac{2}{3} \quad (1)$$

$$2x = 41.81\dots, 138.18\dots$$

$$x = 20.9^\circ, 69.1^\circ \quad (1 \text{ d.p.})$$

$$(1) \quad (1)$$

2.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5 \sin^2 x + 4 \sin x - 5 = 0 \quad (3)$$

(b) Hence solve, for $0 < x \leq 360^\circ$

$$4 \tan x = 5 \cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4 \tan 3x = 5 \cos 3x$$

in the interval $0 < x \leq 1800^\circ$, explaining briefly the reason for your answer.

(2)

$$a) \quad 4 \tan x = 5 \cos x$$

$$4 \frac{\sin x}{\cos x} = 5 \cos x \quad (1)$$

$$4 \sin x = 5 \cos^2 x$$

$$4 \sin x = 5 (1 - \sin^2 x) \quad (1)$$

$$4 \sin x = 5 - 5 \sin^2 x$$

$$5 \sin^2 x + 4 \sin x - 5 = 0 \quad (1)$$

$$b) \quad 4 \tan x = 5 \cos x \quad \equiv \quad 5 \sin^2 x + 4 \sin x - 5 = 0 \quad (1)$$

$$\text{Let } a = \sin x, \quad 5a^2 + 4a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{29}}{5}$$

$$\sin x = \frac{-2 \pm \sqrt{29}}{5} \quad (1)$$

$$\boxed{\sin x = \frac{-2 + \sqrt{29}}{5}}, \quad \frac{-2 - \sqrt{29}}{5}$$

not a solution
✓ because the value is less than -1, so it is outside of range of $\sin x$

$$x = \sin^{-1} \frac{-2 + \sqrt{29}}{5} \quad (1)$$

$$= 42.6^\circ \quad \text{and} \quad 180^\circ - 42.6^\circ$$

$$(1) = 137.4^\circ$$

take the value from 2nd quadrant also since $\sin$ is +ve on this quadrant.

$\sin$	ALL
2nd	1st
3rd	4th
$\tan$	$\cos$

c) for interval of $0^\circ < x \leq 360^\circ$,

The number of solution would be $2 \times 3 = 6$ because the x value decrease by factor of 3. (1)

for interval of $0^\circ < x \leq 1800^\circ$, which is 5 times larger than the previous interval,

the number of solutions would be $6 \times 5 = 30$. (1)