

1. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leq 450^\circ$, the equation

$$5 \cos^2 \theta = 6 \sin \theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

“Solve, for $-90^\circ < x < 90^\circ$, the equation $3 \tan x - 5 \sin x = 0$ ”

is set out below.

$$\begin{aligned} 3 \tan x - 5 \sin x &= 0 \\ 3 \frac{\sin x}{\cos x} - 5 \sin x &= 0 \\ 3 \sin x - 5 \sin x \cos x &= 0 \\ 3 - 5 \cos x &= 0 \\ \cos x &= \frac{3}{5} \\ x &= 53.1^\circ \end{aligned}$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40^\circ) = \frac{3}{5}$$

are $\alpha_1, \alpha_2, \alpha_3$ and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

(i) As $\cos^2 \theta = 1 - \sin^2 \theta$ ①

Then, $5 \cos^2 \theta = 6 \sin \theta$

$5(1 - \sin^2 \theta) = 6 \sin \theta$

$5 - 5 \sin^2 \theta = 6 \sin \theta$

$5 \sin^2 \theta + 6 \sin \theta - 5 = 0$ ①

We can use quadratic formula for $ax^2 + bx + c = 0$,
where $a = 5$, $b = 6$, $c = -5$ and $x = \sin \theta$

This gives us

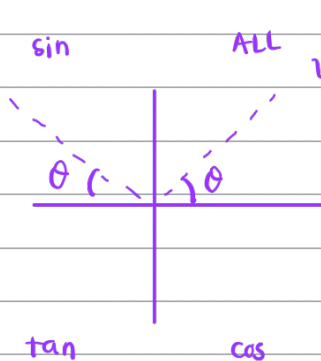
$$\sin \theta = \frac{-6 \pm \sqrt{6^2 - 4(5)(-5)}}{2(5)}$$

we only take the positive root of the quadratic in $\sin \theta$

because negative root

$$\sin \theta = \frac{-6 \pm 2\sqrt{34}}{2(5)} = \frac{-3 \pm \sqrt{34}}{5}$$

① gives no solution as $\sin \theta \neq -ve$ values



$\sin \theta$ can only be positive

$$\theta = 34.484\dots, (180^\circ - 34.484^\circ), (360^\circ + 34.484^\circ)$$

$$\theta = 34.484^\circ, 145.516^\circ, 394.484^\circ$$

$$\theta = 34.5^\circ, 145.5^\circ, 394.5^\circ \text{ (1 d.p.)}$$

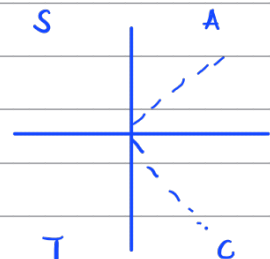
②

(ii) (a) They cancel out $\sin x$ on line 4 and so miss the solution

$$\sin x = 0 \rightarrow x = 0 \text{ ①}$$

They do not find all the solutions to $\cos x = \frac{3}{5}$ in the given range, they miss the solution $x = -53.1^\circ$. ①

CAST diagram



(ii) (b) By a CAST diagram, we know these are the solutions for $\cos(5\alpha + 40) = \frac{3}{5}$

$$\text{at } 5\alpha_1 + 40^\circ = 53.1^\circ$$

$$5\alpha_2 + 40^\circ = 360^\circ - 53.1^\circ$$

$$5\alpha_3 + 40^\circ = 360^\circ + 53.1^\circ$$

$$\Rightarrow 5\alpha_4 + 40^\circ = 720^\circ - 53.1^\circ \text{ ①}$$

$$5\alpha_4 = 720^\circ - 53.1^\circ - 40^\circ = 626.9^\circ$$

$$\alpha_4 = 125^\circ \text{ (nearest degree) ①}$$