Questions

Q1.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leq x \leq 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

(Total for question = 7 marks)

Q2.

Solve, for $360° \leq x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for question = 5 marks)

Q3.

(a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 8 marks)

Q4.

The curve *C*, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve *C* passes through the origin *O*

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for sin 2*y* to find an equation linking *x* and *y* for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (*x*, *y*) lying on *C*,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where *a* and *b* are constants to be found.

(3)

(Total for question = 7 marks)

Q5.

Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

(Total for question = 3 marks)

Q6.

(a) Solve, for $-180° \leq x < 180°$, the equation

$$3 \sin^2 x + \sin x + 8 = 9 \cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9 \cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

(Total for question = 8 marks)

Q7.

Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for question = 3 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\frac{10\sin^2\theta - 7\cos\theta + 2}{3+2\cos\theta} \equiv \frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3+2\cos\theta}$	M1	1.1b
	$\equiv \frac{12 - 7\cos\theta - 10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv \frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4 - 5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4 + 3\sin x = 4 - 5\cos x \Rightarrow \tan x = -\frac{5}{3}$	M1	2.1
	$x = \text{awrt } 121°, 301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta = 1 - \cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\frac{12 - 7\cos\theta - 10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\frac{12-7u-10u^2}{3+2u}$ where they introduce $u = \cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u = \cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2 \leftrightarrow \cos^2\theta$ or $\sin^2 = 1 - \cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\frac{10\sin^2\theta - 7\cos\theta + 2}{3+2\cos\theta} \equiv \frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{} = \frac{12 - 7\cos\theta - 10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x = k, \quad k \neq 0$

Condone $\theta \leftrightarrow x$ Do not condone $a\tan x = 0 \Rightarrow \tan x = b \Rightarrow x = ...$

Alternatively squares $3\sin x = -5\cos x$ and uses $\sin^2 x = 1 - \cos^2 x$ oe to reach $\sin x = A, -1 < A < 1$ or $\cos x = B, -1 < B < 1$

A1: Either $x = \text{awrt } 121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x = \text{awrt } 121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x = \text{awrt } 121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x = \text{awrt } 121°$ or $301°$

Alternative proof in part (a):
M1: Multiplies across and form 3TQ in $\cos\theta$ on rhs
$10\sin^2\theta - 7\cos\theta + 2 = (4-5\cos\theta)(3+2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$

A1: Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$

dM1: Uses $\cos^2\theta = 1-\sin^2\theta$ on the rhs or $\sin^2\theta = 1-\cos^2\theta$ on the lhs
Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe

A1*: Shows that $(4-5\cos\theta)(3+2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true"

Q2.

Question	Scheme	Marks	AOs
	Uses $\sin^2 x = 1-\cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x = 430.5°, 435.5°$	A1	1.1b
			(5 marks)

Notes

M1: Uses correct identity
A1: Correct three term quadratic
M1: Solves their three term quadratic to give values for $\cos x$ – (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)
M1: Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain
A1: Two correct answers in the given domain

Q3.

Question	Scheme	Marks	AOs
(a)	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \dfrac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1 - \cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \dfrac{2}{3}, -\dfrac{1}{2}$	B1	1.1b
	$x = \dfrac{1}{3}\arccos\left(\dfrac{2}{3}\right)$ or $\dfrac{1}{3}\arccos\left(-\dfrac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
		(8 marks)	

Notes

(a)

M1: Recall and use the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **Note that it cannot just be stated.**

A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.

This is scored for a correct line that does not contain any fractional terms.

It may be awarded later in the solution after the identity $1 - \cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2\left(1 - \cos^2\theta\right)$ or equivalent

M1: Attempts to use the correct identity $1 - \cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$

A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

(b)

M1: For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$" where y could be $\cos 3x$, $\cos x$, or even just y. When factorising look for $(ay + b)(cy + d)$ where $ac = \pm 6$ and $bd = \pm 2$

This may be implied by the correct roots (even award for $\left(y \pm \dfrac{2}{3}\right)\left(y \pm \dfrac{1}{2}\right)$), an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even $\pm$ the correct roots.

B1: For the roots $\dfrac{2}{3}, -\dfrac{1}{2}$ oe

M1: Finds at least one solution for x from $\cos 3x$ **within the given range** for their $\dfrac{2}{3}, -\dfrac{1}{2}$

A1: $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

Q4.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	This mark is given for differentiating and inverting
	At (0, 0), $\frac{dy}{dx} = \frac{1}{8}$	A1	This mark is given for finding $\frac{dy}{dx}$ when $y = 0$
(b)(i)	$\sin 2y \approx 2y \Rightarrow x \approx 8y$	B1	This mark is given for finding an approximation for x
(b)(ii)	When x and y are small, $x = 4\sin 2y$ approximates to the line $x = 8y$	B1	This mark is given for a valid explanation of the relationship between x and y when both are small
(c)	$\sin^2 2y + \cos^2 2y = 1$ $\Rightarrow \cos^2 2y = 1 - \sin^2 2y$ $x = 4\sin 2y \Rightarrow \sin^2 2y = \left(\frac{x}{4}\right)^2$	M1	This mark is given for a method to use find an expression for $\sin^2 2y$ in terms of x
	$\frac{dy}{dx} = \frac{1}{8\cos 2y} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$	A1	This mark is given for an unsimplified expression for $\frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	This mark is given for a fully correct answer with $a = 2$ and $b = 16$

Q5.

Question	Scheme	Marks	AOs
	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$= \frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$$\rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$$ and attempts to simplify.

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} = \frac{8\theta^2}{6\theta} = \frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta \sin 3\theta} = \frac{2\sin^2 2\theta}{2\theta \sin 3\theta} = \frac{2\times(2\theta)^2}{2\theta \times 3\theta} = \frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

Q6.

Question	Scheme	Marks	AOs
(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° =$ their '$-19.47°$'

A1ft: $\theta = 5.26°$ but allow a follow through on their '$-19.47°$'

Q7.

Question	Scheme	Marks	AOs
(a)	Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$ It should be $\frac{\sin\theta}{\cos\theta} = \tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°) \neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
			(3 marks)

Notes:

(a)

B1: Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta} = \tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta = \frac{1}{2}$ not $\tan\theta = 2$'

Accept also statements such as 'it should be $\cot\theta = 2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta = 2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°) = +ve$ and $2\sin(-26.6°) = -ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ± 5