Questions

Q1.

Solve, for $360° \leq x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for question = 5 marks)

Q2.

(a) Show that the equation

$$4 \cos \theta - 1 = 2 \sin \theta \tan \theta$$

can be written in the form

$$6 \cos^2\theta - \cos \theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 90°$

$$4 \cos 3x - 1 = 2 \sin 3x \tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 8 marks)

Q3.

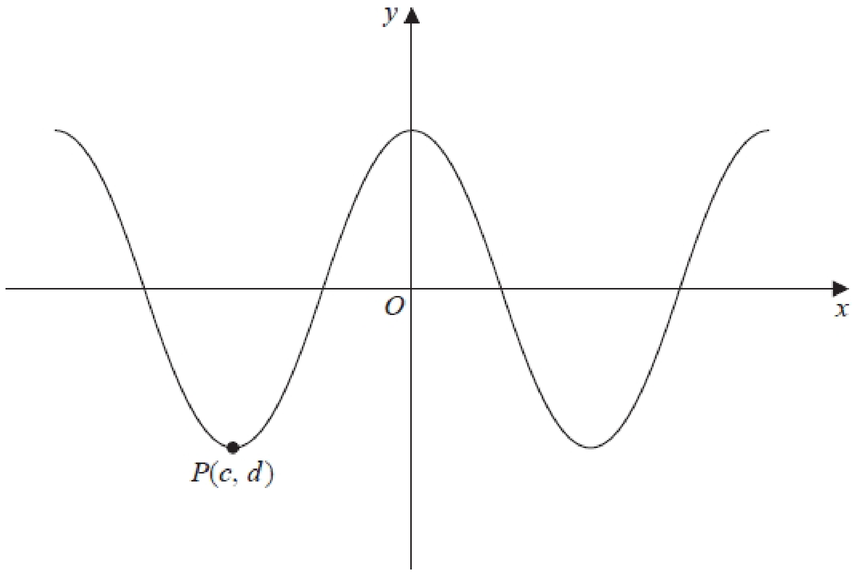

Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which
a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x°$ to the curve with equation

(i) $y = 3 \cos \left(\frac{x°}{4}\right)$
(ii) $y = 3 \cos (x - 36)°$

(2)

(c) Solve, for $450° \leq \theta < 720°$

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

(Total for question = 8 marks)

Q4.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leq 450°$, the equation

$$5 \cos^2\theta = 6 \sin \theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3 \tan x - 5 \sin x = 0$"

is set out below.

$$3 \tan x - 5 \sin x = 0$$
$$3 \frac{\sin x}{\cos x} - 5 \sin x = 0$$
$$3 \sin x - 5 \sin x \cos x = 0$$
$$3 - 5 \cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are α_1, α_2, α_3 and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

(Total for question = 9 marks)

Q5.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leq x \leq 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

(Total for question = 7 marks)

Q6.

$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$

(a) (i) Calculate f (2)

(ii) Write f (x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leq \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

(Total for question = 6 marks)

Q7.

Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos \theta = 2 \sin \theta$ $\tan \theta = 2$ $\theta = 63.4°$	$\cos \theta = 2 \sin \theta$ $\cos^2 \theta = 4 \sin^2 \theta$ $1 - \sin^2 \theta = 4\sin^2 \theta$ $\sin^2 \theta = \frac{1}{5}$ $\sin \theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos \theta = 2 \sin \theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for question = 3 marks)

Q8.

(a) Solve, for $-180° \leq x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

(Total for question = 8 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow\ 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x =\ 430.5°,\ 435.5°$	A1	1.1b
		(5 marks)	

Notes

M1: Uses correct identity
A1: Correct three term quadratic
M1: Solves their three term quadratic to give values for $\cos x$ – (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)
M1: Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain
A1: Two correct answers in the given domain

Q2.

Question	Scheme	Marks	AOs
(a)	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \frac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1-\cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3}\arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3}\arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
		(8 marks)	

Notes

(a)

M1: Recall and use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **Note that it cannot just be stated.**

A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.
This is scored for a correct line that does not contain any fractional terms.
It may be awarded later in the solution after the identity $1-\cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2(1-\cos^2\theta)$ or equivalent

M1: Attempts to use the correct identity $1-\cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$
A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

(b)

M1: For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$ " where y could be $\cos 3x$, $\cos x$, or even just y. When factorsing look for $(ay+b)(cy+d)$ where $ac = \pm 6$ and $bd = \pm 2$
This may be implied by the correct roots (even award for $\left(y \pm \frac{2}{3}\right)\left(y \pm \frac{1}{2}\right)$), an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even $\pm$ the correct roots.

B1: For the roots $\frac{2}{3}, -\frac{1}{2}$ oe

M1: Finds at least one solution for x from $\cos 3x$ **within the given range** for their $\frac{2}{3}, -\frac{1}{2}$

A1: $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

Q3.

Question	Scheme	Marks	AOs
(a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \frac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(°)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \to (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \to (c + 36°, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan\theta = \frac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin\theta$

B1: Uses the correct identity and multiplies across to give $3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$ oe

M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

A1: $\sin\theta = \frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"

A1: Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

Q4.

Question	Scheme	Marks	AOs
(i)	Uses $\cos^2\theta = 1-\sin^2\theta$ $5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin\theta = \dfrac{-3+\sqrt{34}}{5} \Rightarrow \theta = ...$	dM1	3.1a
	$\Rightarrow \theta = 34.5°, 145.5°, 394.5°$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40° = 720° - 53.1°$	M1	3.1a
	$\alpha = 125°$	A1	1.1b
		(2)	
			(9 marks)

Notes

(i)

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to form a 3TQ in $\sin\theta$

A1: Correct 3TQ=0 $5\sin^2\theta + 6\sin\theta - 5 = 0$

dM1: Solves their 3TQ in $\sin\theta$ to produce one value for θ. It is dependent upon having used $\cos^2\theta = \pm 1 \pm \sin^2\theta$

A1: Two of awrt $\theta = 34.5°, 145.5°, 394.5°$ (or if in radians two of awrt 0.60, 2.54, 6.89)

A1: All three of awrt $\theta = 34.5°, 145.5°, 394.5°$ and no other values

(i) (a)

See scheme

(ii)(b)

M1: Sets $5\alpha + 40° = 666.9°$ o.e.

A1: awrt $\alpha = 125°$

Q5.

Question	Scheme	Marks	AOs
(a)	$\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\dfrac{5}{3}$	M1	2.1
	$x=\text{awrt }121°,301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\dfrac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}=\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow\tan x=b\Rightarrow x=\ldots$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2x=1-\cos^2x$ oe to reach $\sin x=A,-1<A<1$ or $\cos x=B,-1<B<1$

A1: Either $x=\text{awrt }121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=\text{awrt }121°$ **or** $301°$

Alternative proof in part (a):
M1: Multiplies across and form 3TQ in $\cos\theta$ on rhs
$10\sin^2\theta - 7\cos\theta + 2 = (4 - 5\cos\theta)(3 + 2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$
A1: Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$
dM1: Uses $\cos^2\theta = 1 - \sin^2\theta$ on the rhs or $\sin^2\theta = 1 - \cos^2\theta$ on the lhs
Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe
A1*: Shows that $(4 - 5\cos\theta)(3 + 2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true"

Q6.

Question	Scheme	Marks	AOs
	(a) $\text{f}(x) = -3x^3 + 8x^2 - 9x + 10,\quad x \in \mathbb{R}$		
(a)	(i) $\{\text{f}(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ \text{f}(2) = 0$	B1	1.1b
	(ii) $\{\text{f}(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \le \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

Notes for Question	
(a)(i)	
B1:	$\mathrm{f}(2)=0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm 3x^2 \pm kx + ...,\ k=$ value $\neq 0$ or $\pm 3x^2 \pm \alpha x + \beta,\ \beta =$ value $\neq 0$, α can be 0 • factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c)$, $k=$ value $\neq 0$, c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta)$, $\beta =$ value $\neq 0$, α can be 0
A1:	$(x-2)(-3x^2+2x-5)$, $(2-x)(3x^2-2x+5)$ or $-(x-2)(3x^2-2x+5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2-4(-3)(-5)$, $4-60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4+2y^2-5$" which would lead to a value <0 together with an explanation that $-3y^4+2y^2-5=0$ has no {real} solutions • or for the omission of <0
Note:	<0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2-4(-3)(-5)=-54<0$
Note:	$y^2=2 \Rightarrow y=\pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4+2y^2-5=0$ or $-3x^2+2x-5=0$ gives y^2 or $x=\dfrac{-2\pm\sqrt{-56}}{-6}$ or $\dfrac{-1\pm\sqrt{-14}}{-3}$
Note:	Completing the square on $-3x^2+2x-5=0$ gives $x^2-\dfrac{2}{3}x+\dfrac{5}{3}=0 \Rightarrow \left(x-\dfrac{1}{3}\right)^2-\dfrac{1}{9}+\dfrac{5}{3}=0 \Rightarrow x=\dfrac{1}{3}\pm\sqrt{\dfrac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta=$ awrt 23.1, awrt 26.2, awrt 29.4 **without** reference to 3 solutions

Q7.

Question	Scheme	Marks	AOs
(a)	Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$ It should be $\frac{\sin\theta}{\cos\theta} = \tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°) \neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
			(3 marks)

Notes:

(a)

B1: Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta} = \tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta = \frac{1}{2}$ not $\tan\theta = 2$'

Accept also statements such as 'it should be $\cot\theta = 2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta = 2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°) = +ve$ and $2\sin(-26.6°) = -ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ± 5

Q8.

Question	Scheme	Marks	AOs
(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9\left(1 - \sin^2 x\right)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° =$ their '$-19.47°$'

A1ft: $\theta = 5.26°$ but allow a follow through on their '$-19.47°$'