Questions

Q1.

Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

(Total for question = 3 marks)

Q2.

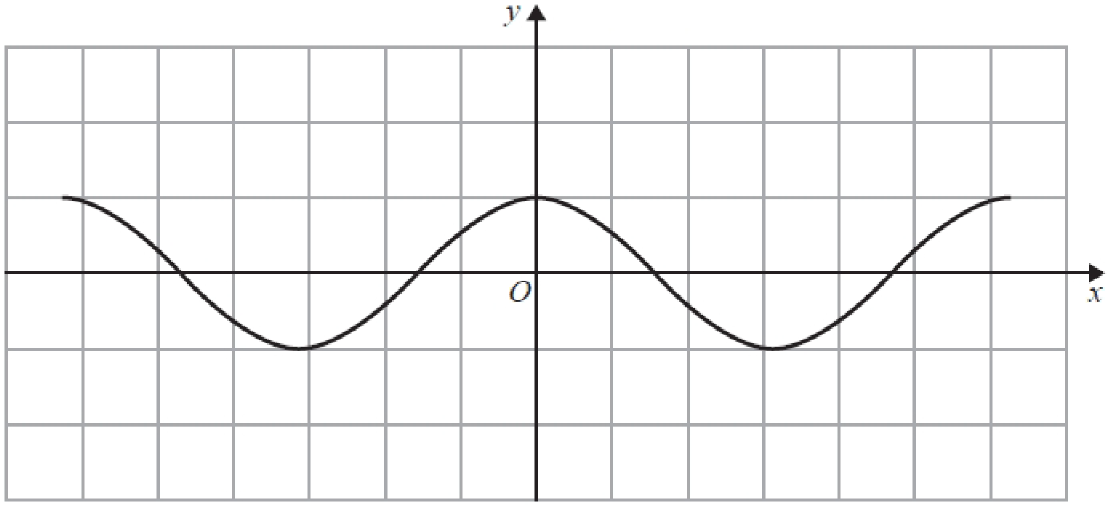


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians.
Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is *a*, and that *a* is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of *a* to 3 decimal places.

(3)

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

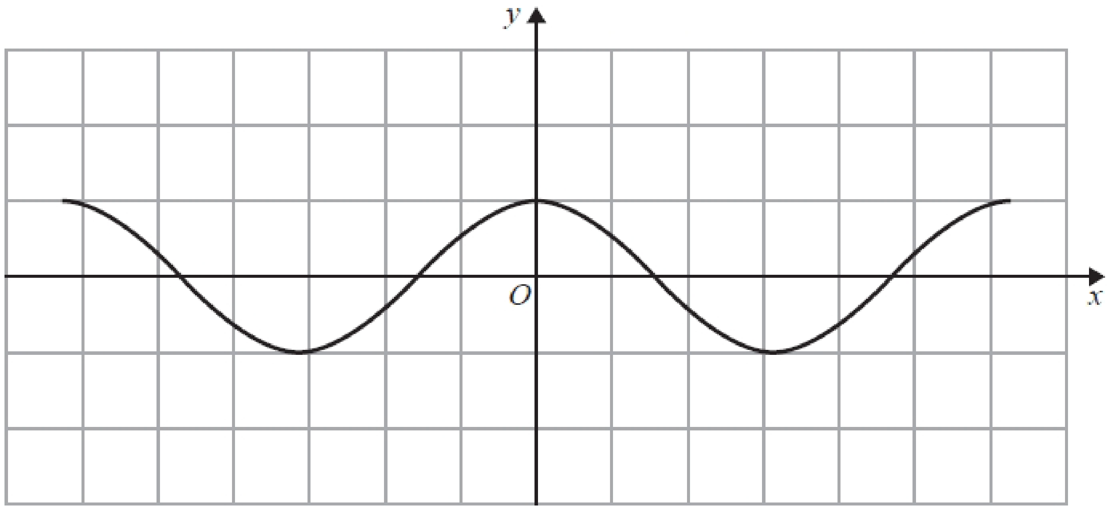


Diagram 1

....

....

....

....

....

....

....

....

....

....

....

....

....

(Total for question = 5 marks)

Q3.

The curve *C*, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve *C* passes through the origin *O*

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for sin 2*y* to find an equation linking *x* and *y* for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (*x*, *y*) lying on *C*,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where *a* and *b* are constants to be found.

(3)

(Total for question = 7 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$\rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify.

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}=\frac{8\theta^2}{6\theta}=\frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

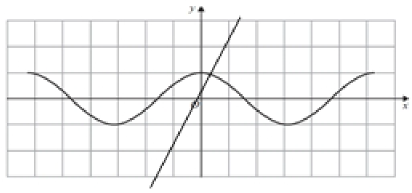
Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}=\frac{1-(1-2\sin^2 2\theta)}{2\theta \sin 3\theta}=\frac{2\sin^2 2\theta}{2\theta \sin 3\theta}=\frac{2\times(2\theta)^2}{2\theta \times 3\theta}=\frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

Q2.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for plotting the line $y = 2x + \frac{1}{2}$ on the diagram with a correct gradient and intercept
	Only one intersection means that there is one root	A1	This mark is given for a reason why there is only one real root
(b)	$1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	This mark is given for using the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation
	$x^2 + 4x - 1 = 0$	M1	This mark is igvne for rearranging to find a quadratic equation to solve
	0.236 or $-2 + \sqrt{5}$	A1	This mark is given for finding the correct (positive) solution for x

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	This mark is given for differentiating and inverting
	At (0, 0), $\frac{dy}{dx} = \frac{1}{8}$	A1	This mark is given for finding $\frac{dy}{dx}$ when $y = 0$
(b)(i)	$\sin 2y \approx 2y \Rightarrow x \approx 8y$	B1	This mark is given for finding an approximation for x
(b)(ii)	When x and y are small, $x = 4\sin 2y$ approximates to the line $x = 8y$	B1	This mark is given for a valid explanation of the relationship between x and y when both are small
(c)	$\sin^2 2y + \cos^2 2y = 1$ $\Rightarrow \cos^2 2y = 1 - \sin^2 2y$ $x = 4\sin 2y \Rightarrow \sin^2 2y = \left(\frac{x}{4}\right)^2$	M1	This mark is given for a method to use find an expression for $\sin^2 2y$ in terms of x
	$\frac{dy}{dx} = \frac{1}{8\cos 2y} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$	A1	This mark is given for an unsimplified expression for $\frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	This mark is given for a fully correct answer with $a = 2$ and $b = 16$