Questions

Q1.

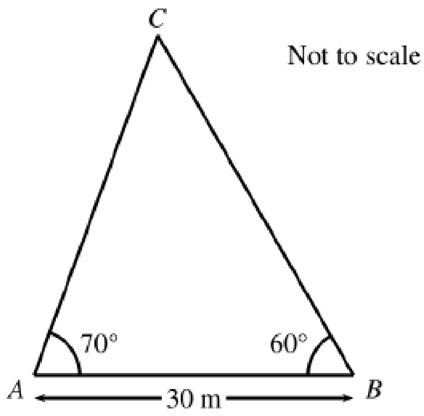


Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 70° and angle *ABC* = 60°,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

(Total for question = 5 marks)

Q2.

In a triangle *ABC*, side *AB* has length 10 cm, side *AC* has length 5 cm, and angle *BAC* = θ where θ is measured in degrees. The area of triangle *ABC* is 15cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that *BC* is the longest side of the triangle,

(b) find the exact length of *BC*.

(2)

(Total for question = 6 marks)

Q3.

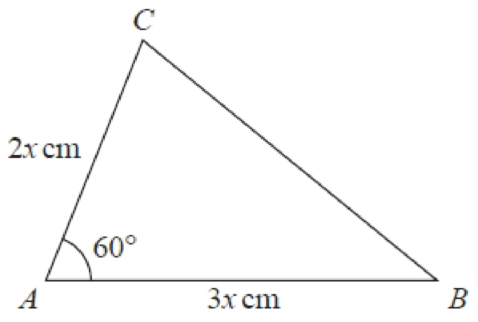


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

(Total for question = 6 marks)

Q4.

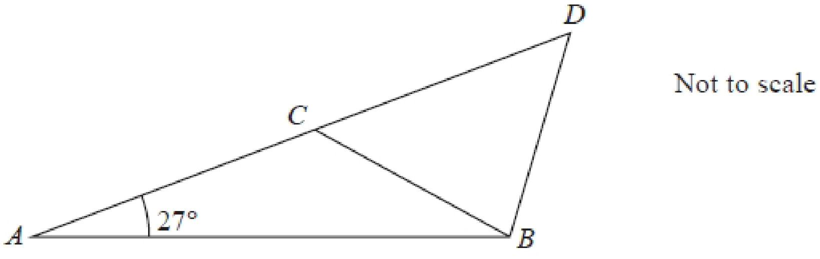


Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, *AB*, *BD*, *BC* and *AD*.

Given *AB* = 12m, *BC* = *BD* = 7m and angle *BAC* = 27°

(a) find, to one decimal place, the size of angle *ACB*.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

(Total for question = 6 marks)

Q5.

A parallelogram *PQRS* has area 50 cm^2

Given

- *PQ* has length 14 cm
- *QR* has length 7 cm
- angle *SPQ* is obtuse

find

(a) the size of angle *SPQ*, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal *SQ*, in cm, to one decimal place.

(2)

(Total for question = 5 marks)

Q6.

[*In this question the unit vectors* **i** and **j** *are due east and due north respectively*.]

A coastguard station *O* monitors the movements of a small boat.

At 10:00 the boat is at the point (4**i** − 2**j**) km relative to *O*.

At 12:45 the boat is at the point (−3**i** − 5**j**) km relative to *O*.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

(Total for question = 6 marks)

Mark Scheme

Q1.

Question	Scheme		Marks	AOs
(a)	Finds third angle of triangle and uses or states $\frac{x}{\sin 60°} = \frac{30}{\sin "50°"}$	Finds third angle of triangle and uses or states $\frac{y}{\sin 70°} = \frac{30}{\sin "50°"}$	M1	2.1
	So $x = \frac{30\sin 60°}{\sin 50°}$ $(= 33.9)$	So $y = \frac{30\sin 70°}{\sin 50°}$ $(= 36.8)$	A1	1.1b
	Area = $\frac{1}{2} \times 30 \times x \times \sin 70°$ or $\frac{1}{2} \times 30 \times y \times \sin 60$		M1	3.1a
	$= 478\text{ m}^2$		A1ft	1.1b
			(4)	
(b)	Plausible reason e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat		B1	3.2b
			(1)	
				(5 marks)

Notes

(a) M1: Uses sine rule with their third angle to find one of the unknown side lengths
A1: finds expression for, or value of either side length
M1: Completes method to find area of triangle
A1ft: Obtains a correct answer for their value of x or their value of y.

(b) B1: As information given in the question may not be accurate to 4sf or the lawn may not be flat so modelling by a plane figure may not be accurate.

Q2.

Question	Scheme	Marks	AOs
(a)	Uses $15=\frac{1}{2}\times5\times10\times\sin\theta$	M1	1.1b
	$\sin\theta=\frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta=1-\sin^2\theta$	M1	2.1
	$\cos\theta=\pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2=10^2+5^2-2\times10\times5\times"-\frac{4}{5}"$	M1	3.1a
	$BC=\sqrt{205}$	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

M1: Uses the formula Area $=\frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or θ

A1: $\sin\theta=\frac{3}{5}$ oe This may be implied by $\theta=$ awrt $36.9°$ or awrt 0.644 (radians)

M1: Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta=1-\sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or candidates may just write down the **two values**. The values must be symmetrical $\pm k$

A1: $\cos\theta=\pm\frac{4}{5}$ or ±0.8 Condone these values appearing from $\pm0.79....$

(b)

M1: Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find BC using the cosine rule. Alternatively works out BC using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

A1: $BC=\sqrt{205}$

Q3.

Question	Scheme	Marks	AOs
(a)	Uses $18\sqrt{3}=\frac{1}{2}\times 2x\times 3x\times\sin 60°$	M1	1.1a
	Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $x^2=k$ oe	M1	1.1b
	$x=\sqrt{12}=2\sqrt{3}$ *	A1*	2.1
		(3)	
(b)	Uses $BC^2=\left(6\sqrt{3}\right)^2+\left(4\sqrt{3}\right)^2-2\times 6\sqrt{3}\times 4\sqrt{3}\times\cos 60°$	M1	1.1b
	$BC^2=84$	A1	1.1b
	$BC=2\sqrt{21}$ (cm)	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts to use the formula $A=\frac{1}{2}ab\sin C$.

If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times\sin 60°$ **without** sight of a previous correct line then this would be M0

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$

This may be awarded from the correct formula or $A=ab\sin C$

A1*: Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$

This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$

Alternative using the given answer of $x=2\sqrt{3}$

M1: Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$

A1*: Concludes that $x=2\sqrt{3}$

(b)

M1: Attempts the cosine rule with the sides in the correct position.

This can be scored from $BC^2=(3x)^2+(2x)^2-2\times 3x\times 2x\times\cos 60°$ as long as there is some attempt to substitute x in later. Condone slips on the squaring

A1: $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$

If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0

A1: $BC=2\sqrt{21}$

Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

Q4.

Question	Scheme	Marks	AOs
(a)	States $\frac{\sin\theta}{12}=\frac{\sin 27}{7}$	M1	1.1b
	Finds $\theta=$ awrt $51°$ or awrt $129°$	A1	1.1b
	$=$ awrt $128.9°$	A1	1.1b
		(3)	
(b)	Attempts to find part or all of AD Eg $AD^2=7^2+12^2-2\times12\times7\cos101.9=(AD=15.09)$ Eg $(AC)^2=7^2+12^2-2\times12\times7\cos(180-\text{"}128.9\text{"}-27)$ Eg $12\cos27$ or $7\cos\text{"}51\text{"}$	M1	1.1b
	Full method for the total length $=12+7+7+\text{"}15.09\text{"}=$	dM1	3.1a
	$=42$ m	A1	3.2a
		(3)	
			(6 marks)

Notes

(a)

M1: States $\frac{\sin\theta}{12}=\frac{\sin 27}{7}$ oe with the sides and angles in the correct positions

Alternatively they may use the cosine rule on $\angle ACB$ and then solve the subsequent quadratic to find AC and then use the cosine rule again

A1: awrt $51°$ or awrt $129°$

A1: Awrt $128.9°$ only (must be seen in part a))

(b)

M1: Attempts a "correct" method of finding either AD or a part of AD eg (AC or CD or forming a perpendicular to split the triangle into two right angled triangles to find AX or XD) which may be seen in (a).
You should condone incorrect labelling of the side.
Look for attempted application of the cosine rule

$$(AD)^2=7^2+12^2-2\times12\times7\cos(\text{"}128.9\text{"}-27)$$

$$\text{or } (AC)^2=7^2+12^2-2\times12\times7\cos(180-\text{"}128.9\text{"}-27)$$

Or an attempted application of the sine rule $\frac{(AD)}{\sin(\text{"}128.9\text{"}-27)}=\frac{7}{\sin27}$

$$\text{Or } \frac{(AC)}{\sin(180-\text{"}128.9\text{"}-27)}=\frac{7}{\sin27}$$

Or an attempt using trigonometry on a right-angled triangle to find part of AD

$12\cos 27$ or $7\cos"51"$

This method can be implied by sight of awrt 15.1 or awrt 6.3 or awrt 8.8 or awrt 10.7 or awrt 4.4

dM1: A complete method of finding the TOTAL length.
There must have been an attempt to use the correct combination of angles and sides.
Expect to see $7+7+12+"AD"$ found using a correct method.
This is scored by either $7+7+12+"AD"$ if $\angle ACB = 128.9°$ in a) or
$7+7+12+\text{awrt}15.1$ by candidates who may have assumed $\angle ACB = 51.1°$ in a)

A1: Rounds correct 41.09 m (or correct expression) up to 42 m to find steel **bought**

Candidates who assumed $\angle ACB = 51.1°$ (acute) in (a):
Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 15.1$ so ignore any labelling of the lengths they are finding.

Diagram of the correct triangle with lengths and angles:

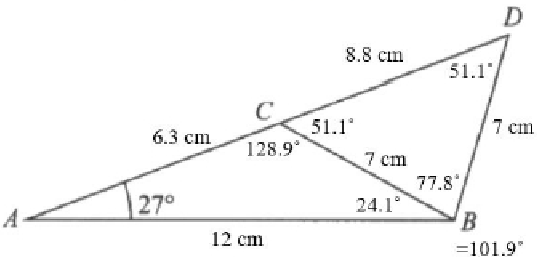
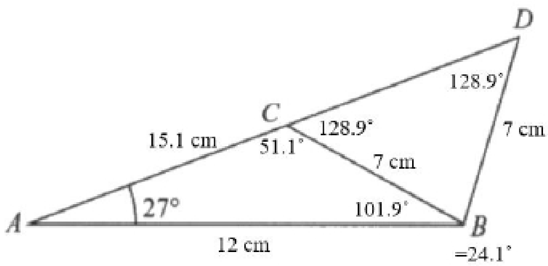


Diagram using the incorrect acute angle:



Q5.

Question	Scheme	Marks	AOs
(a)	Sets $50 = 7\times 14\sin(SPQ)$ oe	B1	1.2
	Finds $180° - \arcsin\left("\frac{50}{98}"\right)$	M1	1.1b
	$= 149.32°$	A1	1.1b
		(3)	
(b)	Method of finding SQ $SQ^2 = 14^2 + 7^2 - 2\times 14\times 7\cos"149.32"$	M1	1.1b
	$= 20.3$ cm	A1	1.1b
		(2)	
			(5 marks)
Alt(a)	States or uses $14h = 50$ or $7h_1 = 50$	B1	1.2
	Full method to find obtuse $\angle SPQ$. In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$	M1	1.1b
	awrt $149.32°$	A1	1.1b

Notes

(a)

B1: Sets $50 = 7\times14\sin(SPQ)$ oe

M1: Attempts the correct method of finding obtuse $\angle SPQ$. See scheme.

A1: awrt 149.32°

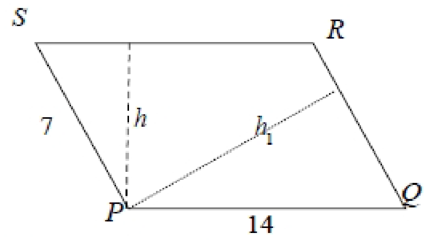
(b)

M1: A correct method of finding SQ using their $\angle SPQ$.

$SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$ scores this mark.

A1: awrt 20.3 cm (condone lack of units)

Alt(a)



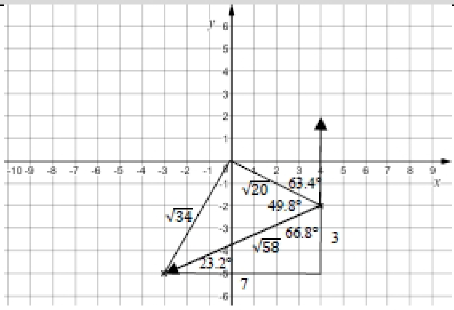
B1: States or uses $14h = 50$ or $7h_1 = 50$

M1: Full method to find obtuse $\angle SPQ$.

In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$

A1: awrt 149.32°

Q6.

Question	Scheme	Marks	AOs
(a)			
	Attempts to find an "allowable" angle Eg $\tan\theta = \frac{7}{3}$	M1	1.1b
	A full attempt to find the bearing Eg $180° + "67°"$	dM1	3.1b
	Bearing = awrt $246.8°$	A1	1.1b
		(3)	
(b)	Attempts to find the distance travelled = $\sqrt{(4--3)^2+(-2+5)^2} = (\sqrt{58})$	M1	1.1b
	Attempts to find the speed = $\frac{\sqrt{58}}{2.75}$	dM1	3.1b
	= awrt 2.77 km h^{-1}	A1	1.1b
		(3)	
			(6 marks)

Notes: Score these two parts together.

(a)

M1: Attempts an allowable angle. (Either the "66.8", "23.2" or ("49.8" and "63.4"))

$\tan\theta = \pm\frac{7}{3}$, $\tan\theta = \pm\frac{3}{7}$, $\tan\theta = \pm\frac{-2--5}{4--3}$ etc

There must be an attempt to subtract the coordinates (seen or applied at least once)

If part (b) is attempted first, look for example for $\sin\theta = \pm\frac{7}{"\sqrt{58}"}$, $\cos\theta = \pm\frac{7}{"\sqrt{58}"}$, etc

They may use the cosine rule and trigonometry to find the two angles in the scheme. See above. Eg award for $\cos\theta = \frac{"58"+"20"-"34"}{2\times"\sqrt{58}"\times"\sqrt{20}"}$ **and** $\tan\theta = \pm\frac{4}{2}$ or equivalent.

dM1: A full attempt to find the bearing. $180° + \arctan\frac{7}{3}$, $270° - \arctan\frac{3}{7}$, $360° - "49.8°" - "63.4°"$. It is dependent on the previous method mark.

A1: Bearing = awrt $246.8°$ oe. Allow S $66.8°$ W

(b)

M1: Attempts to find the distance travelled. Allow for $d^2 = (4 - -3)^2 + (-2+5)^2$

You may see this on a diagram and allow if they attempt to find the magnitude from their "resultant vector" found in part (a).

dM1: Attempts to find the speed. There must have been an attempt to find the distance using the coordinates and then divide it by 2.75. Alternatively they could find the speed in km min^{-1} and then multiply by 60

A1: awrt 2.77 km h^{-1}