1 (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

a) $LHS = \dfrac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$

$= \dfrac{12 - 10\cos^2\theta - 7\cos\theta}{3 + 2\cos\theta}$

$= \dfrac{-[10\cos^2\theta + 7\cos\theta - 12]}{3 + 2\cos\theta}$

$= \dfrac{-[(5\cos\theta - 4)\cancel{(2\cos\theta + 3)}]}{\cancel{(3 + 2\cos\theta)}}$

$= 4 - 5\cos\theta = RHS$

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Question 1 continued

b) we can instead solve:

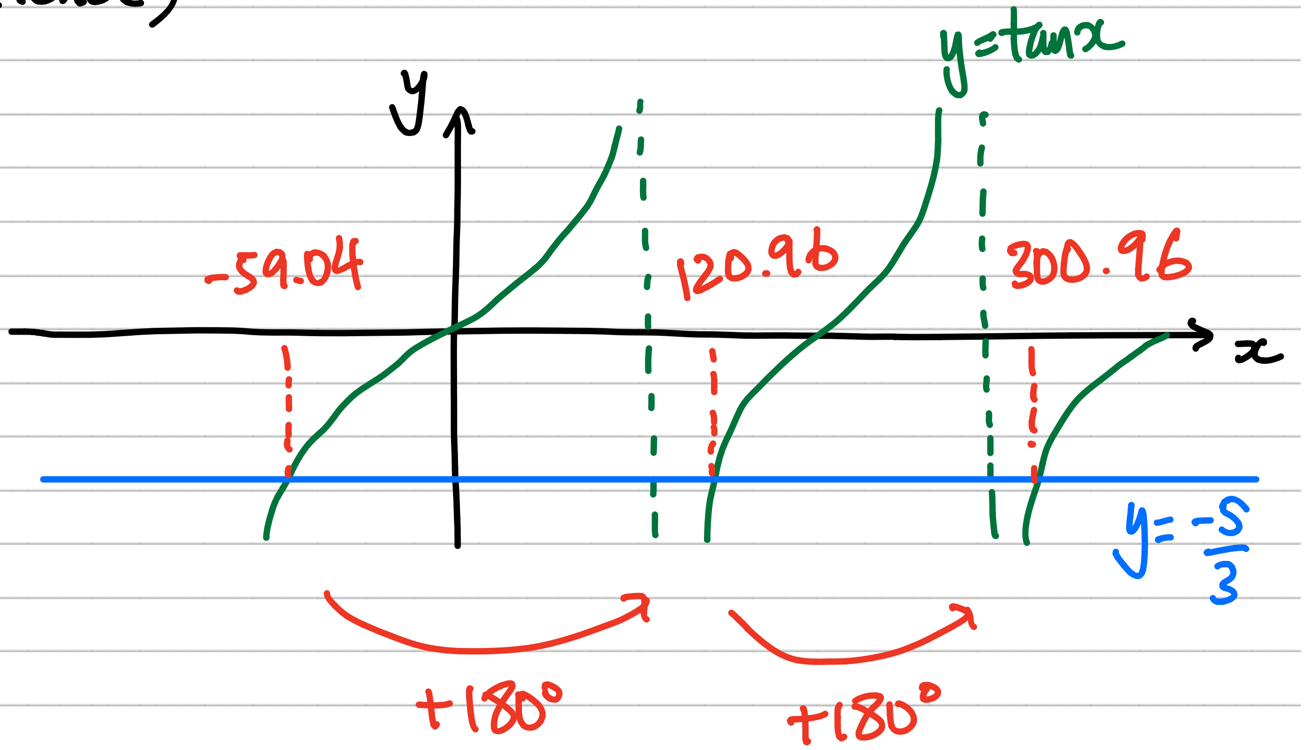
$$\cancel{4} - 5\cos x = \cancel{4} + 3\sin x$$

$$\sin x = -\frac{5}{3}\cos x$$

$\div \cos x$: $\tan x = -\frac{5}{3}$

$$x = \tan^{-1}\left(-\frac{5}{3}\right) = -59.04^\circ$$

Hence,



$\therefore x = \underline{\underline{121^\circ}}, \underline{\underline{301^\circ}}$

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2. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Since $\sin^2 x \equiv 1 - \cos^2 x$,

$$12(1-\cos^2 x) + 7\cos x - 13 = 0$$

$$12 - 12\cos^2 x + 7\cos x - 13 = 0$$

$$-1 - 12\cos^2 x + 7\cos x = 0$$

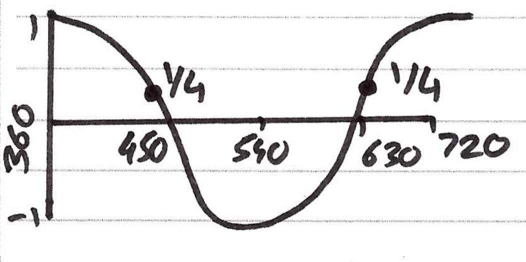
$$12\cos^2 x - 7\cos x + 1 = 0$$

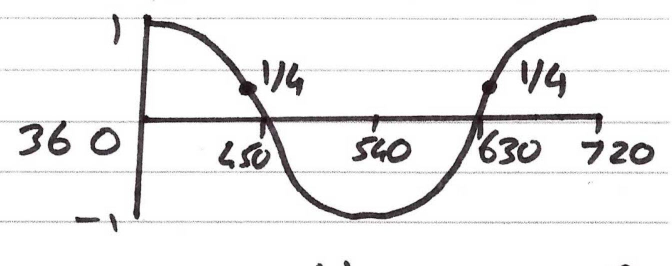
$$(4\cos x - 1)(3\cos x - 1) = 0$$

Either $4\cos x - 1 = 0$ or $3\cos x - 1 = 0$

$4\cos x = 1 \qquad 3\cos x = 1$

$\cos x = \frac{1}{4} \qquad \cos x = \frac{1}{3}$





$\cos^{-}(\frac{1}{4}) = 75.5°$ $\qquad \cos^{-1}(\frac{1}{3}) = 70.5°$

In given domain, $x = 360 + 75.5$ or $360 + 70.5°$

$$\boxed{x = 430.5°, 435.5° \text{ (to 1d.p.)}}$$

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3. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

a) $4\cos\theta - 1 = 2\sin\theta\tan\theta$

$= 2\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{2\sin^2\theta}{\cos\theta}$

$= \frac{2(1-\cos^2\theta)}{\cos\theta}$

$= \frac{2}{\cos\theta} - 2\cos\theta$

$6\cos\theta - 1 - \frac{2}{\cos\theta} = 0$

$\times\cos\theta \quad 6\cos^2\theta - \cos\theta - 2 = 0$

b) $4\cos 3x - 1 = 2\sin x\tan 3x$

$6\cos^2(3x) - \cos 3x - 2 = 0$

$(3\cos 3x - 2)(2\cos 3x + 1) = 0$

$\cos 3x = \frac{2}{3} \qquad \cos 3x = \frac{-1}{2}$

$3x = 48.19° \qquad 3x = 120°, 240°$

$x = 16.1° \qquad x = 40°, 80°$

4. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

a) $x = 4\sin 2y$

$\frac{dx}{dy} = 4(2\cos 2y)$ 1

$= 8\cos 2y$

Take reciprocal

$\frac{dy}{dx} = \frac{1}{8\cos 2y}$

At origin $(0,0)$ so sub $y=0$

$\frac{dy}{dx} = \frac{1}{8\cos(0)}$

$\cos(0) = 1$

$\frac{dy}{dx} = \frac{1}{8}$ 1

small angle approximation

bi) $\sin x \approx x$

$\sin 2y \approx 2y$ 1

$\therefore x = 4\sin 2y$

using $\sin 2y \approx 2y$

$x \approx 4(2y)$

$x \approx 8y$

bii) Value found in a) is the gradient of the line found in bi) 1

$y = \frac{1}{8}x$

can see by re-arranging that gradient same as value in a)

c) $\frac{dy}{dx} = \frac{1}{8\cos 2y}$

$x = 4\sin 2y$

$\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 2y + \cos^2 2y = 1$

$x^2 = 16\sin^2 2y$

using $\sin^2 2y = 1 - \cos^2 2y$ (1)

$x^2 = 16(1 - \cos^2 2y)$

$x^2 = 16 - 16\cos^2 2y$ (1)

$16\cos^2 2y = 16 - x^2$

$\cos^2 2y = 1 - \frac{x^2}{16}$

$\cos 2y = \sqrt{1 - \frac{x^2}{16}}$

$\frac{dy}{dx} = \frac{1}{8\sqrt{1 - \frac{x^2}{16}}} \begin{matrix} \times\sqrt{16} \\ \times\sqrt{16} \end{matrix}$

$= \frac{\cancel{\sqrt{16}}\ 4}{8\sqrt{16 - x^2}}$

$= \frac{1}{2\sqrt{16 - x^2}}$ (1)

5. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

$\cos\theta \approx 1 - \frac{\theta^2}{2}$ $\quad$ $\sin\theta \approx \theta$

$\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ ✓ $\quad$ $\sin 3\theta \approx 3\theta$

$= 1 - 8\theta^2$

$\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-(1-8\theta^2)}{2\theta \, 3\theta}$

$= \frac{1-1+8\theta^2}{6\theta^2}$ ✓

$= \frac{8\theta^2}{6\theta^2} = \frac{8}{6} = \frac{4}{3}$ ✓

6. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

a) $\sin^2 x + \cos^2 x \equiv 1$

$\cos^2 x \equiv 1 - \sin^2 x$

$3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$ - 1

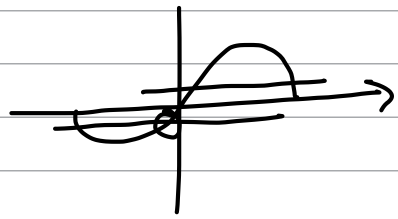
$3\sin^2 x + \sin x + 8 = 9 - 9\sin^2 x$

$12\sin^2 x + \sin x - 1 = 0$ - 1

$12\sin^2 x + 4\sin x - 3\sin x - 1 = 0$

$4\sin x(3\sin x + 1) - (3\sin x + 1) = 0$

$(3\sin x + 1)(4\sin x - 1) = 0$ - 1



$3\sin x + 1 = 0$ $\qquad$ $4\sin x - 1 = 0$

$\sin x = -\frac{1}{3}$ - 1 - $\sin x = \frac{1}{4}$

$x = -19.47°, -160.53°$ $\qquad$ $x = 14.48°, 165.52°$ - 1

$x = -160.53°, -19.47°, 14.48°, 165.52°$ (2 d.p.) - 1

b) $2\theta - 30° = -160.53°$ ✗

$2\theta - 30° = -19.47°$ - 1

$2\theta = 10.53°$

$\theta = 5.26°$ - 1

7. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos \theta = 2 \sin \theta$ $\tan \theta = 2$ $\theta = 63.4°$	$\cos \theta = 2 \sin \theta$ $\cos^2 \theta = 4 \sin^2 \theta$ $1 - \sin^2 \theta = 4\sin^2 \theta$ $\sin^2 \theta = \frac{1}{5}$ $\sin \theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

$\sin^2\theta + \cos^2\theta \equiv 1$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos \theta = 2 \sin \theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

a) $\frac{\cos\theta}{\sin\theta} = \tan\theta$

It should be $\frac{\sin\theta}{\cos\theta} = \tan\theta$

b) i) $\cos(-26.6) = 0.894$

$2\sin(-26.6) = -0.896$

$\cos\theta \neq 2\sin\theta$

$\therefore$ answer is incorrect.

ii) the incorrect answer arose from squaring.