1. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

Since $\sin^2 x \equiv 1 - \cos^2 x$,

$$12(1-\cos^2 x) + 7\cos x - 13 = 0$$

$$12 - 12\cos^2 x + 7\cos x - 13 = 0$$

$$-1 - 12\cos^2 x + 7\cos x = 0$$

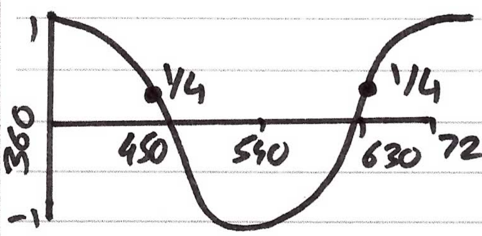
$$12\cos^2 x - 7\cos x + 1 = 0$$

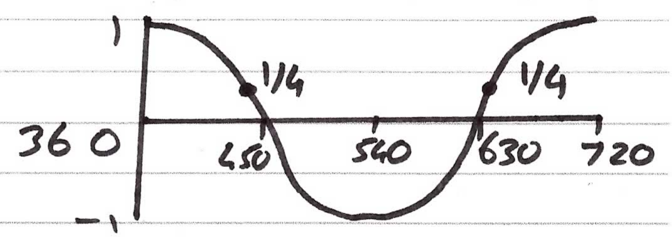
$$(4\cos x - 1)(3\cos x - 1) = 0$$

Either $4\cos x - 1 = 0$ or $3\cos x - 1 = 0$

$4\cos x = 1$ $\quad$ $3\cos x = 1$

$\cos x = \frac{1}{4}$ $\quad$ $\cos x = \frac{1}{3}$





$\cos^{-}(\frac{1}{4}) = 75.5°$ $\qquad$ $\cos^{-1}(\frac{1}{3}) = 70.5°$

In given domain, $x = 360 + 75.5$ or $360 + 70.5°$

$$\boxed{x = 430.5°, \ 435.5° \quad (\text{to } 1\text{d.p.})}$$

2. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

2a) $4\cos\theta - 1 = 2\sin\theta\tan\theta$

$= 2\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{2\sin^2\theta}{\cos\theta}$

$= \frac{2(1-\cos^2\theta)}{\cos\theta}$

$= \frac{2}{\cos\theta} - 2\cos\theta$

$6\cos\theta - 1 - \frac{2}{\cos\theta} = 0$

$\times\cos\theta$ $\quad 6\cos^2\theta - \cos\theta - 2 = 0$

b) $4\cos 3x - 1 = 2\sin x\tan 3x$

$6\cos^2(3x) - \cos 3x - 2 = 0$

$(3\cos 3x - 2)(2\cos 3x + 1) = 0$

$\cos 3x = \frac{2}{3}$ $\qquad$ $\cos 3x = \frac{-1}{2}$

$3x = 48.19°$ $\qquad$ $3x = 120°, 240°$

$x = 16.1°$ $\qquad$ $x = 40°, 80°$

3.

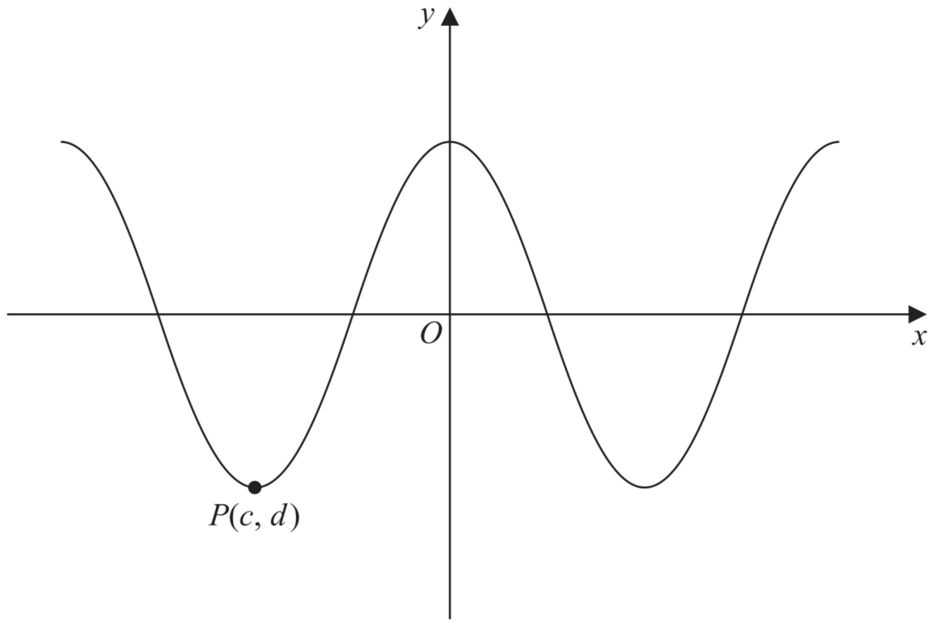


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

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Question continued

a) minimum of $\cos x = -1 \Rightarrow$ minimum of $3\cos x = -3 = d$

P is the first minimum for $x<0$ $\therefore$ $c = -180°$

$P(-180°, -3)$

b) i. $y = 3\cos\left(\frac{x°}{4}\right) \rightarrow$ x-'stretched' x 4, no change to y

$\hookrightarrow (-720°, -3)$

ii. $y = 3\cos(x - 36°) \rightarrow$ translation in x-direction $+36°$

$\hookrightarrow (-144°, -3)$

c) When solving these types of question, list the relevant trig. identities that could help you.

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta \equiv 1$

$3\cos\theta = 8\tan\theta$

$= 8\frac{\sin\theta}{\cos\theta}$

$\times \cos\theta$: $3\cos^2\theta = 8\sin\theta$

Form quadratic in $\sin\theta$ (you want an equation with only one trig. function)

$3(1 - \sin^2\theta) = 8\sin\theta$

Question continued

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

$$(3\sin\theta - 1)(\sin\theta + 3) = 0$$

$\sin\theta \neq -3$ so $\sin\theta = \frac{1}{3}$

always note what range you need

min. value is -1

$\theta = 19.47^\circ$

but range is $450^\circ \leq \theta < 720^\circ$

$\theta = 180 - 19.47^\circ$

sinθ repeats every 360°

$= 160.53^\circ$

to bring into range: $160.53^\circ + 360^\circ = \underline{520.5^\circ}$ (1 d.p.)

$19.47^\circ + 360^\circ$ also not in range

In this question you should show all stages of your working.

4.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are α_1, α_2, α_3 and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

Question continued

(i) $5\cos^2\theta = 6\sin\theta$

$5(1-\sin^2\theta) = 6\sin\theta$ (1)

$5 - 5\sin^2\theta = 6\sin\theta$

$5\sin^2\theta + 6\sin\theta - 5 = 0$ (1)

use calculator to get values of $\sin\theta$

$\sin\theta = 0.56619..., -1.76619...$

$\sin\theta \neq -1.76619...$

$\sin\theta = 0.56619...$ (1)

$\theta = 34.48°, 145.51°, 394.48°$

$\therefore \theta = 34.5°, 145.5°, 394.5°$ (1) (1) (1dp) ※

(ii) (a)

Line 4 : They cancel by $\sin x$, and hence they miss the solution $\sin x = 0$, which is $x = 0$. (1)

Line 6 : They do not find all the solutions for $\cos x = \frac{3}{5}$ in the given range $-90° < x < 90°$. So, they miss the solution $x = -53.1°$ ※ (1)

Question continued

(ii) (b)

$\cos(5\alpha + 40) = \frac{3}{5}$

1

$5\alpha + 40 = 53.13\ldots, 306.869\ldots, 413.13\ldots, 666.869\ldots$

$\alpha = \frac{53.13\ldots - 40}{5}, \frac{306.869\ldots - 40}{5}, \frac{413.13\ldots - 40}{5}, \frac{666.869\ldots - 40}{5}$

$\alpha = 2.62\ldots, 53.37\ldots, 74.62\ldots, 125.37\ldots$

$(\alpha_1) \quad (\alpha_2) \quad (\alpha_3) \quad (\alpha_4)$

$\therefore \alpha_4 = 125°$ (nearest degree) #

1

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5. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

a) $\text{LHS} = \dfrac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$

$= \dfrac{12 - 10\cos^2\theta - 7\cos\theta}{3 + 2\cos\theta}$

$= \dfrac{-[10\cos^2\theta + 7\cos\theta - 12]}{3 + 2\cos\theta}$

$= \dfrac{-[(5\cos\theta - 4)\cancel{(2\cos\theta + 3)}]}{\cancel{(3 + 2\cos\theta)}}$

$= 4 - 5\cos\theta = \text{RHS}$

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Question continued

b) we can instead solve:

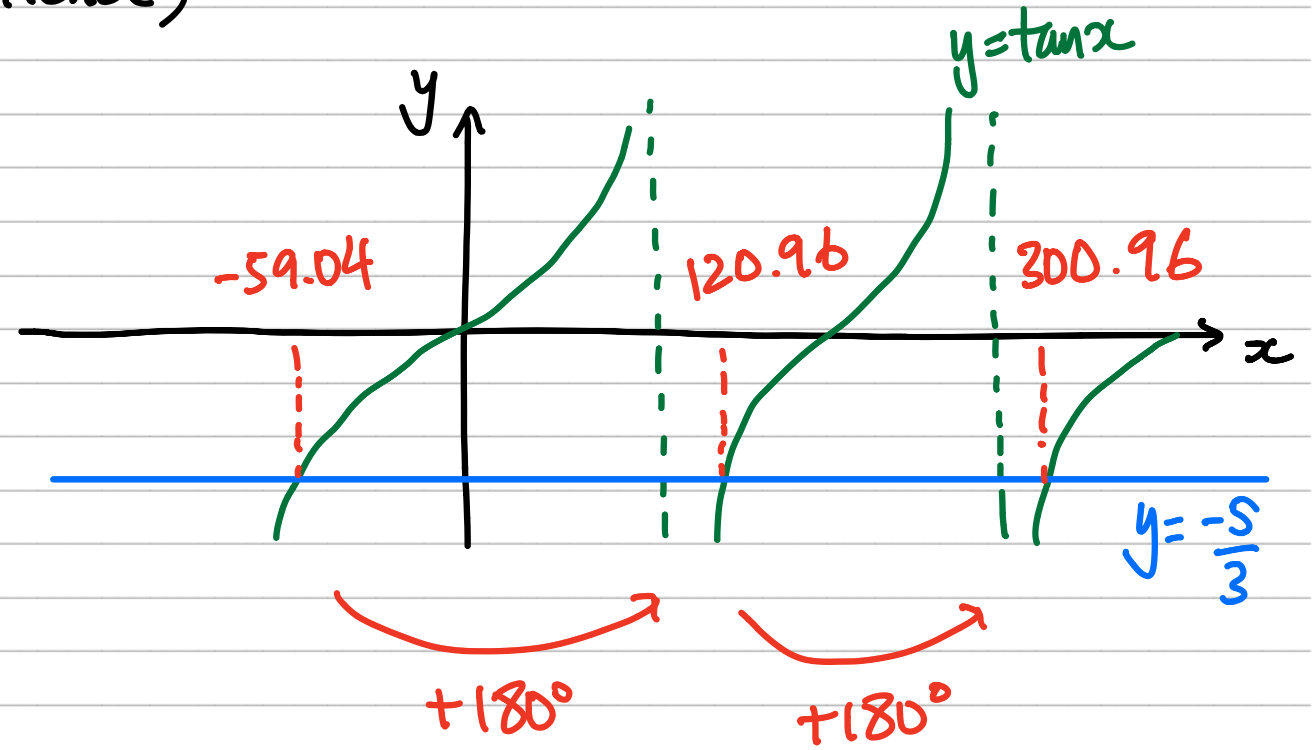
$$\cancel{4} - 5\cos x = \cancel{4} + 3\sin x$$

$$\sin x = -\frac{5}{3}\cos x$$

$\div \cos x$: $\tan x = -\frac{5}{3}$

$$x = \tan^{-1}\left(-\frac{5}{3}\right) = -59.04°$$

Hence,



$\therefore x = \underline{\underline{121°}}, \underline{\underline{301°}}$

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

ai) $f(2) = -3(2)^3 + 8(2)^2 - 9(2) + 10 = \underline{\underline{0}}$ ✓1

ii) If $x = 2$ gives $f(2) = 0$, then $(x-2)$ is a factor by the factor theorem.

$f(x) = (x-2)(ax^2 + bx + c) = -3x^3 + 8x^2 - 9x + 10$

$\underline{x^3:}$ $a = -3$

$\therefore a = -3$

$\underline{x^2:}$ $b - 2a = 8$

$b = 8 + 2a$

$b = 8 + 2(-3) = 2$

$\underline{\text{const.:}}$ $-2c = 10$

$c = -5$

$f(x) = \underline{(x-2)(-3x^2 + 2x - 5)}$ ✓1 ✓1

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

b) $f(x) = -3x^3 + 8x^2 - 9x + 10$
$= (x-2)(-3x^2 + 2x - 5)$

$-3y^6 + 8y^4 - 9y^2 + 10 = 0$

Use the sub : $x = y^2$

$-3x^3 + 8x^2 - 9x + 10 = 0$

$(x-2)(-3x^2 + 2x - 5) = 0$

$x = 2$

$y^2 = 2$

$y = \pm\sqrt{2}$

only two real solutions.

$b^2 - 4ac < 0$

$(2)^2 - 4(-3)(-5) < 0$

$4 - 60 < 0$

$-56 < 0$

$\therefore$ there are no real solutions to this quadratic.

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

c) $f(x) = -3x^3 + 8x^2 - 9x + 10$

Use the sub.: $x = \tan(\theta)$

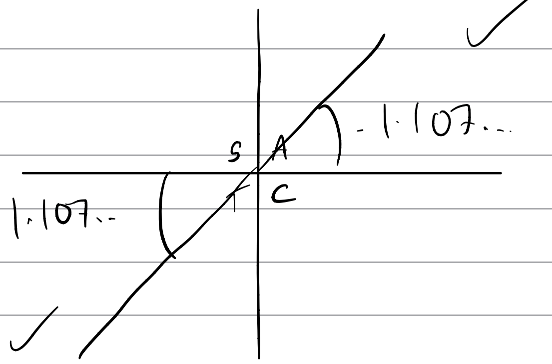
$3x^3 - 8x^2 + 9x - 10 = 0$

$-3x^3 + 8x^2 - 9x + 10 = 0$

From earlier parts: $x = 2$ is the only solution

$\tan(\theta) = 2$

$\theta = 1.107...$



No. of solutions in $7\pi \leq \theta \leq 10\pi$

is the same in $0 \leq \theta \leq 3\pi$

$\theta = 1.107...,\ 1.107... + \pi,\ 1.107... + 2\pi$

$\Rightarrow \therefore$ there are 3 solutions for θ.

7. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

<u>Student *A*</u>	<u>Student *B*</u>
$\cos\theta = 2\sin\theta$	$\cos\theta = 2\sin\theta$
$\tan\theta = 2$	$\cos^2\theta = 4\sin^2\theta$
$\theta = 63.4°$	$1 - \sin^2\theta = 4\sin^2\theta$
	$\sin^2\theta = \frac{1}{5}$
	$\sin\theta = \pm\frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

$\sin^2\theta + \cos^2\theta \equiv 1$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

a) $\frac{\cos\theta}{\sin\theta} = \tan\theta$

It should be $\frac{\sin\theta}{\cos\theta} = \tan\theta$

b) i) $\cos(-26.6) = 0.894$

$2\sin(-26.6) = \cdot 0.896$

$\cos\theta \neq 2\sin\theta$

$\therefore$ answer is incorrect.

ii) the incorrect answer arose from squaring.

8. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

a) $\sin^2 x + \cos^2 x \equiv 1$

$\cos^2 x \equiv 1 - \sin^2 x$

$3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$ – 1

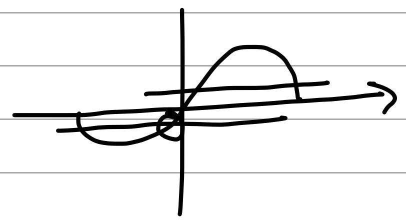
$3\sin^2 x + \sin x + 8 = 9 - 9\sin^2 x$

$12\sin^2 x + \sin x - 1 = 0$ – 1

$12\sin^2 x + 4\sin x - 3\sin x - 1 = 0$

$4\sin x(3\sin x + 1) - (3\sin x + 1) = 0$

$(3\sin x + 1)(4\sin x - 1) = 0$ – 1



$3\sin x + 1 = 0$ $\quad$ $4\sin x - 1 = 0$

$\sin x = -\frac{1}{3}$ – 1 – $\sin x = \frac{1}{4}$

$x = -19.47°, -160.53°$ $\quad$ $x = 14.48°, 165.52°$ – 1

$x = -160.53°, -19.47°, 14.48°, 165.52°$ (2 d.p.) – 1

b) $\cancel{2\theta - 30° = -160.53°}$

$2\theta - 30° = -19.47°$ – 1

$2\theta = 10.53°$

$\theta = 5.26°$ – 1