1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

$\cos\theta \approx 1 - \frac{\theta^2}{2}$ $\qquad$ $\sin\theta \approx \theta$

$\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ ✓ $\qquad$ $\sin 3\theta \approx 3\theta$

$= 1 - 8\theta^2$

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-(1-8\theta^2)}{2\theta\, 3\theta}$$

$$= \frac{1 - 1 + 8\theta^2}{6\theta^2} \checkmark$$

$$= \frac{8\theta^2}{6\theta^2} = \frac{8}{6} = \frac{4}{3} \checkmark$$

2.

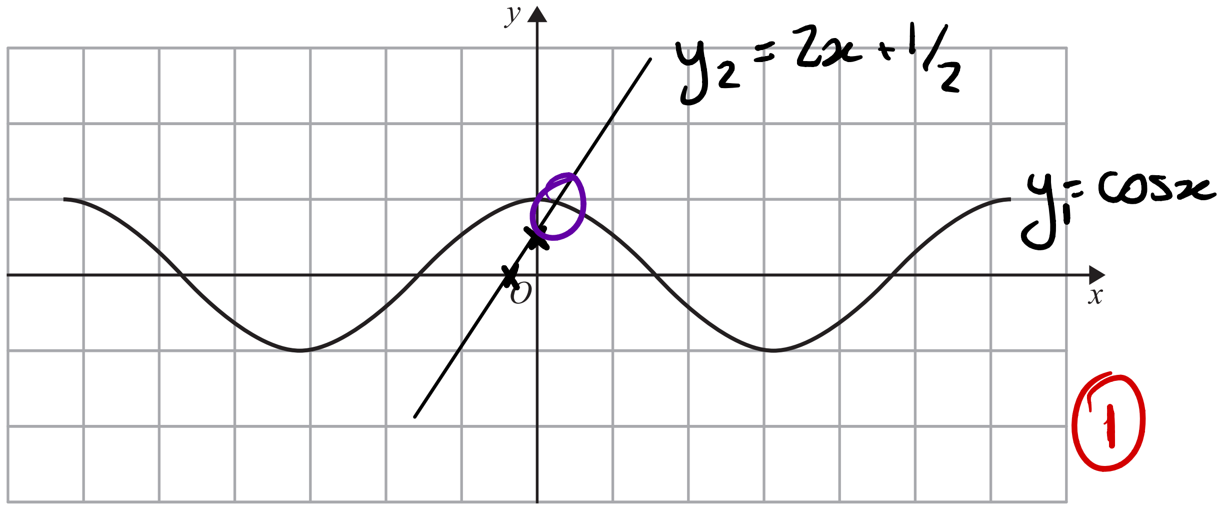

Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

$\cos x = 2x + 1/2$

$y_1 = \cos x$

$y_2 = 2x + 1/2$

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

a) $y_2 = 0$

$0 = 2x + 1/2$

$2x = -1/2$

$x = -1/4$

Since there is only one point of intersection between the functions $y_1 = \cos x$ and $y_2 = 2x + 1/2$ it follows the equation $\cos x - 2x - 1/2 = 0$ has only one real root

1

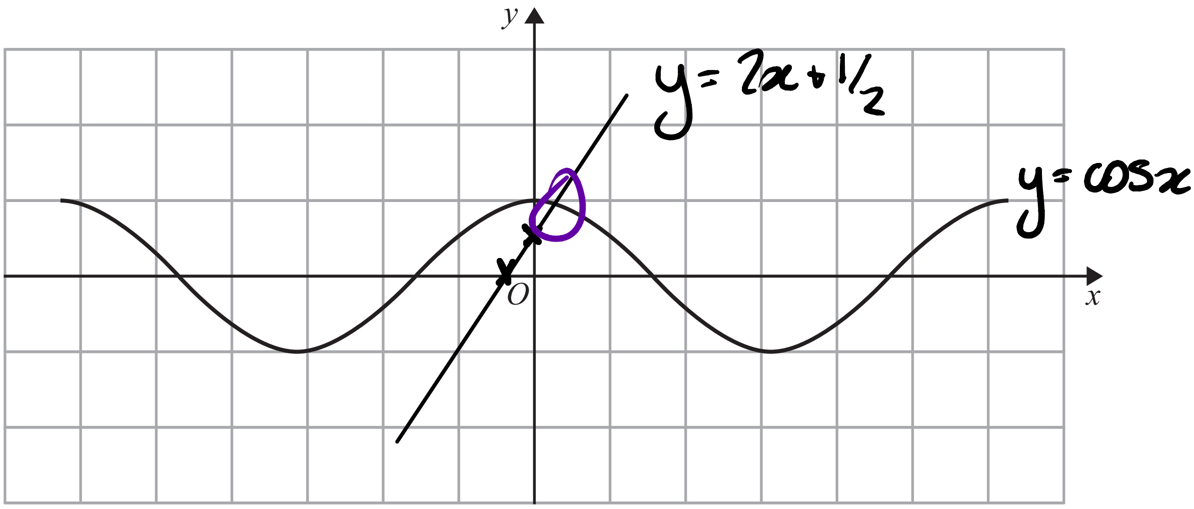

Diagram 1

b) $\cos x = 1 - \frac{1}{2}x^2$

sub into 'original equation'

$1 - \frac{1}{2}x^2 - 2x - \frac{1}{2} = 0$ (1)

$-\frac{1}{2}x^2 - 2x + \frac{1}{2} = 0$

$\times -2$

$x^2 + 4x - 1 = 0$ (1)

$x = -2 + \sqrt{5}$

$x = -2 - \sqrt{5}$ x reject

$\therefore \alpha \approx 0.236$ (1)

3. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

a) $x = 4\sin 2y$

$\frac{dx}{dy} = 4(2\cos 2y)$ (1)

$= 8\cos 2y$

Take reciprocal

$\frac{dy}{dx} = \frac{1}{8\cos 2y}$

At origin $(0,0)$ so sub $y=0$

$\frac{dy}{dx} = \frac{1}{8\cos(0)}$

$\cos(0) = 1$

$\frac{dy}{dx} = \frac{1}{8}$ (1)

small angle approximation

bi) $\sin x \approx x$

$\sin 2y \approx 2y$ (1)

$\therefore x = 4\sin 2y$

using $\sin 2y \approx 2y$

$x \approx 4(2y)$

$x \approx 8y$

bii) Value found in a) is the gradient of the line found in bi) (1)

$y = \frac{1}{8}x$

can see by re-arranging that gradient same as value in a)

c) $\frac{dy}{dx} = \frac{1}{8\cos 2y}$

$x = 4\sin 2y$

$\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 2y + \cos^2 2y = 1$

$x^2 = 16\sin^2 2y$

using $\sin^2 2y = 1 - \cos^2 2y$ (1)

$x^2 = 16(1 - \cos^2 2y)$

$x^2 = 16 - 16\cos^2 2y$ (1)

$16\cos^2 2y = 16 - x^2$

$\cos^2 2y = 1 - \frac{x^2}{16}$

$\cos 2y = \sqrt{1 - \frac{x^2}{16}}$

$\frac{dy}{dx} = \frac{1}{8\sqrt{1 - \frac{x^2}{16}}} \times \frac{\sqrt{16}}{\sqrt{16}}$

$= \frac{\cancel{\sqrt{16}}\ 4}{8\sqrt{16 - x^2}}$

$= \frac{1}{2\sqrt{16 - x^2}}$ (1)